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The role of inter vivos financing in the intergenerational transfer of the corporate farm under uncertainty

David Lee Reinders
Iowa State University

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**THE ROLE OF INTER VIVOS FINANCING IN THE INTERGENERATIONAL
TRANSFER OF THE CORPORATE FARM UNDER UNCERTAINTY**

Iowa State University

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**The role of inter vivos financing
in the intergenerational transfer of the
corporate farm under uncertainty**

by

David Lee Reinders

**A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY**

**Department: Economics
Major: Agricultural Economics**

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1983

TABLE OF CONTENTS

	Page
CHAPTER I. INTRODUCTION	1
Purpose of this Study	4
Outline of the Study	6
CHAPTER II. INCORPORATING RISK IN THE DECISION MAKING PROCESS	8
Risk Versus Uncertainty	8
The Decision Making Process	9
Alternate Criteria of Choice	11
Attitudes Towards Risk	23
Mean-Variance Analysis	29
Stochastic Dominance	32
Expected Utility Measured in Terminal Wealth	33
CHAPTER III. STOCHASTIC DOMINANCE	37
Stochastic Dominance Theorems	37
Special Properties of Stochastic Dominance	51
Alternate Approaches to Stochastic Dominance	54
CHAPTER IV. THE THEORY OF THE FIRM	57
Decision Making from a Whole Farm Perspective	57
Flow of Funds Within the Firm	60
The Components of Risk Which Comprise Total Enterprise Risk	65
Theory of Capital Structure	73
Conceptual Considerations for the Type of Financing	98

CHAPTER V. DEVELOPMENT OF THE EMPIRICAL MODEL AND DATA REQUIREMENTS	129
The Empirical Model	130
The Environment and Data Requirements	156
Data Requirements	172
CHAPTER VI. EMPIRICAL FINDINGS	189
Financing Situations Selected for Analysis	189
Illustrative Results	194
Ownership Mix	201
Salaries Versus Dividends	210
Common Stock Versus a Constant Principal Loan at Market Rate of Interest	217
Common Stock Versus a Constant Principal Loan at Below Market Rate of Interest	235
Common Stock Versus a Market Rate Bond	239
Common Stock Versus a Below-Market Rate Bond	240
Constant Principal Loans Versus Bonds	241
Market Versus Below Market Interest Rates	243
CHAPTER VII. SUMMARY AND CONCLUSIONS	250
Suggestions for Further Research	254
Implications	256
BIBLIOGRAPHY	258
ACKNOWLEDGMENTS	268
APPENDIX A: PROOFS OF STOCHASTIC DOMINANCE THEOREMS	269
APPENDIX B: ESTIMATED COEFFICIENTS AND STATISTICAL RESULTS	278

APPENDIX C: THE IOWA STATE UNIVERSITY BUSINESS AND FINANCIAL PLANNING MODEL	292
APPENDIX D: TESTS FOR SIGNIFICANT DIFFERENCES BETWEEN MEANS	303

CHAPTER I. INTRODUCTION

During the 1970s, the combination of rapidly appreciating land values with the trend toward fewer but larger farms dramatically increased the potential equity drain due to estate taxation. When the continuation of the farm business is an objective of the parents, there is an increasing interest to begin the intergenerational transfer process during the parents' lifetimes instead of at their deaths. The focus of this dissertation is upon the role of equity and nonequity financing within the corporate structure as a means of facilitating the intergenerational transfer of the family farm prior to death.

The desire of the parents to continue the life cycle of the farm beyond one generation is not enough. There must be a son, daughter or other family member who wants to manage the farm operation. The on-farm heir must have the skills and been given the opportunity and responsibility to make management decisions prior to the parents' deaths. The size of the farm is also important since it must be large enough to provide a reasonable standard of living for the succeeding family member and his or her family. Finally, continuity beyond one generation depends on the type of transfer plan and the number of on-farm and off-farm heirs involved. As a result of these considerations, it is often difficult to provide for the continuation of many farms beyond one generation. But for those family farms where continuity is desired and feasible, there is an increasing trend to initiate the transfer process as early as possible.

One approach to this problem is to transfer asset ownership by gift or sale from parents to the heir. But the transfer of the asset entails the loss of control of that asset to the parents. A preferred approach is to gift or sell a minority interest in the farm to the heir while maintaining sufficient control of the farm assets to generate adequate retirement income. In addition, the parents may wish to reduce their management responsibilities and increase the responsibilities of the heir so that the heir can gain experience in managing the business. Finally, in inflationary periods, the parents may wish to "freeze" the value of their interest in the farm to mitigate the potential liquidity drain from estate taxation.

The corporate form of business organization is the most conducive accomplishing these objectives. Although the same outcomes can be achieved with the sole proprietorship and partnership, the corporation provides the greatest flexibility in structuring and carrying out the intergenerational transfer process. Within a corporate organizational form, an equity interest in the farm can be transferred to the heir in the form of shares of common stock. An investor interest is accomplished with intrafamily loans or bonds. The interest payments, along with dividends, salaries and directors' fees, can be used to provide the parents and the heir with adequate incomes. But in a world of uncertainty, common stock, loans, bonds and income sharing plans may not always produce their intended effects for both the parents and the heir.

The appropriate framework within which to evaluate equity and non-equity financing of the business and its transfer is whole farm planning

under uncertainty. Whole farm planning in agriculture requires the joint awareness and integration of the production, investment and finance microtheories. The production microtheory addresses the questions of which commodities to produce in what quantities using which technologies. The investment microtheory addresses the issue of which assets to acquire to provide the inputs needed in the production process. The finance microtheory answers the question of how the needed assets should be acquired.

Furthermore, any analysis of whole farm decision making which assumes the decision maker knows with certainty the outcome of his decision at the time he makes it introduces a degree of artificiality into the analysis. To be sure, many worthwhile inferences about economic behavior have been developed under the assumption of certainty. However, the present trend is toward modeling economic behavior in an uncertain environment. Not only does this provide the model with more realism, the results are more applicable to real world situations.

There are a number of reasons why uncertainty is likely to become more important in agriculture in the future. Agriculture has been characterized by rapid technological change over the last two decades. Typically, this new technology is capital intensive, which requires more expensive and more specialized (and therefore less liquid) assets in the production process. This phenomenon increases the fixed operating costs of the farm and thereby increases the firm's exposure to operating risk. A heavier reliance on purchased inputs makes agricultural enterprises more susceptible to price changes in other sectors of the economy. High

interest rates, combined with the increased volatility in the national money markets, have increased the fixed financing costs and the financial risk of farming (78, 124). In addition, agriculture is also evidencing a trend toward lower and more variable profit margins (70, 90, and 124). This trend leads to lower but more variable after-tax cash flows.

Inflation as a cause of uncertainty has become of particular concern during the last decade. Inflationary expectations have been bid into land prices (35, 60 and 74) and interest rates. Inflation, at least in the short run, reduces farm income and liquidity while increasing capital gains and leads to greater income instability (78).

In addition to these causes of uncertainty, a number of other causes always have influenced and will continue to influence decision making in agriculture. These sources include weather and climate variability, the biological processes inherent in agriculture, inaccurate and incomplete information, uncertain product demand, uncertain government programs, and government intervention. As a result, the farm firm must operate within an ever more uncertain environment, and it is within this uncertain environment that intergenerational transfers must be accomplished.

Purpose of this Study

The purpose of this study is to address the issue of facilitating the intergenerational transfer of the family farm where a relevant goal is continuing the business beyond one generation. For a father and mother who wish to bring their son or daughter into the farming operation, there exist a multitude of equity and nonequity financing

methods which will facilitate the transfer process during the lives of the parents. These financing methods will ultimately vest control of the farm in the heir. The options available are limited by only the imagination and creativeness of the parties involved. But it is not clear, in an uncertain world, as to the costs and benefits of various financing arrangements to the parents or to the heir.

In particular, the inter vivos effects of selected financing arrangements are analyzed with the Iowa State University Business and Financial Planning Model for representative farms operating in an inflationary and uncertain environment. Each representative farm is assumed to be incorporated with only one on-farm heir. The financing arrangement is selected and implemented at the beginning of the planning process. It can not be changed to another financing arrangement during the planning horizon. The financing arrangements selected for study are the following:

1. Common Stock

In this situation the parents transfer an equity interest in the farm firm with shares of common stock. Initial ownership patterns of 100 percent held by the parents, 80 percent held by the parents and 60 percent held are analyzed for each of four representative farm sizes. Therefore, the son (or daughter) is assumed to initially own 0, 20, or 40 percent of the common stock.

2. Loans

Constant principal loans at the market rate of interest and three percent below the market rate are analyzed as an alternative to the ownership of common stock by the parents and by the child.

3. Bonds

An interest only loan with principal paid in one balloon payment at maturity is also analyzed. The rate of interest are

set at market and at three percent below market. These are analyzed for both the parents and the child.

Closely related to the financing arrangements are income sharing plans. The following income-sharing plans are analyzed:

1. Dividends

Dividends on common stock are a vehicle for distributing the firm's past and present earnings to the family members in relation to their ownership.

2. Salaries and directors' fees

Salaries and directors' fees can be used to distribute earnings in relation to contributed labor and management skills.

3. Interest payments

Interest payments on intrafamily loans and bonds can be used to distribute the firm's earnings in relation to investor interests in the firm.

Outline of the Study

The following outline is used in this study. The tasks listed below are in the sequence in which they appear in the following chapters:

1. Although there is a consensus that risk should be incorporated into the decision making process, there is no consensus on the best method of incorporation. Chapter II reviews several methods often used to handle risk and develops the maximization of expected utility as the best method.

2. Because the exact specification of the utility function is not known (and, for the sake of generality, it need not be known), stochastic dominance is developed in Chapter III as the means for choosing the

financing arrangement which maximizes expected utility when only certain characteristics are known about the shape of the utility function.

3. The theory of the firm under risk as an integrated whole of the production, investment and finance microtheories is developed in Chapter IV. The components which make up total business risk based on these three microtheories are presented.

This discussion of the theory of the firm also includes a review of the role of equity and nonequity financing.

4. The empirical model, necessary data requirements and representative farms are presented in Chapter V.

5. In Chapter VI, selected equity and nonequity financing arrangements are analyzed for the representative farms with the Iowa State University Business and Financial Planning Model.

6. Chapter VII presents the results and conclusions of this study and suggestions for further research.

CHAPTER II. INCORPORATING RISK IN THE DECISION MAKING PROCESS

Much of economic theory was developed under the assumption that the decision maker knew the outcome of his decision at the time the decision was made. Although this led to many worthwhile inferences about economic behavior, a certain degree of artificiality is present in these analyses.

Over the last twenty years, there has been a growing emphasis on incorporating risk in decision making situations to more realistically model economic behavior. Although the consensus is that risk should be included in economic modeling, there is not general agreement on the best procedure for doing so.

Risk Versus Uncertainty

Until recently, researchers consistently distinguished between risk and uncertainty. This distinction was first proposed by Frank Knight.¹

1. Risk refers to a situation in which several events or outcomes are possible. The probability of each event occurring is known and can be expressed as an objective value.
2. Uncertainty refers to a similar situation except that the numerical probabilities of the different outcomes cannot be specified.

¹ This definition is restated by Friedman (44, p. 282).

Statistically, a risky prospect is one for which the parameters of the probability distribution are known, while uncertainty implies that the parameters are unknown.

Although this distinction was adhered to in the past, it no longer is. Rothschild and Stiglitz (107, p. 225) use the terms interchangeably. Friedman (44, p. 282) contends that the distinction between risk and uncertainty is no longer valid because the decision maker forms his own subjective probabilities when information is not available. The fact that these personal, subjective probabilities may not agree with those of other individuals is not important. What is important is that the individual acts as if the probabilities are known. Following Friedman, no distinction between the terms will be made in this study and the terms will be used interchangeably.

The Decision Making Process

By definition, a decision involves a choice among alternative courses of action offering different consequences (134, p. 4). The decision maker selects one course of action from all of the strategies available to him. In its simplest form, the decision process can be portrayed as in Figure 1. Each A_i in Figure 1 represents a course of action or strategy of which there are n available to the decision maker. Each S_j represents an outcome state or state of nature which may prevail with probability of occurrence P_j . In total, there are m possible outcome states. The consequences (usually measured in monetary

		Outcome States				
		S_1	S_2	·	·	S_m
Courses of Action	A_1	C_{11}	C_{12}	·	·	C_{1m}
	A_2	C_{21}	C_{22}	·	·	·
	·	·	·	·	·	·
	·	·	·	·	·	·
	A_n	C_{n1}	C_{n2}	·	·	C_{nm}
		P_1	P_2	·	·	P_m

Figure 1. A decision table

terms) of selecting strategy A_i when outcome state S_j prevails is indicated by C_{ij} . If only one column in Figure 1 is associated with a nonzero probability (i.e., only one S_j can occur), then the decision maker is said to know with certainty the outcome of each strategy before the choice is made. In this case, the decision maker chooses the strategy that produces the most desirable consequence. However, if it is possible for more than one outcome to prevail (more than one outcome state has a nonzero probability of occurrence), then the decision maker is uncertain as to the ultimate consequence of any course of action. Now the decision maker must employ some criterion to determine the most desirable course of action.

Alternate Criteria of Choice

There is no consensus as to what the best criteria of choice should be, although several criteria have received extensive treatment in economic research. The expected value, safety first, utility maximization, expected utility maximization, mean-variance and stochastic dominance criteria will briefly be reviewed in the remainder of this chapter. Although no attempt is made to make this list all inclusive, these do represent the most popular and widely used criteria.

Expected value criteria

From economic theory, a profit-maximizing firm will equate its marginal revenue from production with its marginal cost of production.

In a world of certainty with well defined production and cost functions, this is nothing more than an exercise in algebra. But for a perfectly competitive firm operating in an uncertain environment, the price and quantity of output will not be known at the time the decision is made on how much to produce. The decision maker, if adhering to the expected value criteria, would use the expected price and expected quantity in his decision process and give no recognition to the probability distribution of expected price or expected quantity.

If the ultimate price and quantity which prevail are the same as the expectation, then the decision maker has equated marginal revenue with marginal cost. However, if the ultimate price and quantity which prevail are something other than those expected, the decision maker has made a less than optimal decision. The reason the expected value criteria produces suboptimal results is that it gives no recognition to the possibility of more than one outcome occurring.

The shortcomings of the expected value criteria have been known for quite some time. In 1732, Daniel Bernoulli (11, pp. 23-26) used a fictitious game, the St. Petersburg Paradox, to illustrate that the expected value criteria could not be representative of rational human behavior. In Bernoulli's game, a fair coin is tossed until a head appears. If a head appears on the n th toss, the player receives 2^n dollars. As an alternative to playing the game, the player is offered a finite sum of money, such as one hundred dollars. The player must decide whether to accept the finite sum or toss the coin until a head appears and then

receive 2^n dollars. Under the expected value criteria, the expected value of the St. Petersburg Paradox is infinite, as shown in Equation 2-1.

$$EV = \sum_{n=1}^{\infty} \frac{1}{2^n} 2^n = \infty. \quad (2-1)$$

But when people were offered a choice between playing the game and a finite sum, they opted for the finite sum. Bernoulli concluded that expected value is not applicable in explaining the behavior of people who act rationally. Bernoulli (11, p. 24) argues that expected value is a function of price which is the same for all persons, but the value of an item to a person is the utility that it yields. Furthermore, Bernoulli hypothesized that utility is a function of existing wealth and that increases in wealth result in increases in utility, but at a decreasing rate. It was not until 1947 that Bernoulli's hypotheses were put on a rigorous foundation.

Safety-first criteria

There are a number of variants which fall within the collective name of safety-first criteria. Basically, all safety-first variants place the top priority upon survival (3, p. 88). They require that the actual payoff exceed some minimum critical level with some specified probability. The decision maker's basic strategy is to avoid outcomes which would result in personal disaster. If the individual, after assuring the strategy chosen will not lead to disaster, tries to maximize some gain,

then the ordering of strategies becomes lexicographic. In this approach, a prospect with a smaller chance of failure is always preferred to a prospect with a larger chance of failure even if the latter has a much higher expected value of gain.

Chance constrained programming (116, pp. 134-140) and focus of loss (17) are examples of safety-first variants. The analysis of particular situations, such as beginning farmers with a very small equity base, may suggest that survival is the paramount goal to satisfy; in these situations, safety-first may be the appropriate method. However, a more general approach is needed because the safety-first criteria focuses attention on the left-most tail of the distribution and ignores the right-tail (83, pp. 41-42). Only in the special case of two prospects with identical chances of failure will the safety-first rule and the expected value rule coincide (3, p. 88).

Utility maximization

The concept of utility is nothing more than the association of an uncertain outcome with a real number in such a fashion that the larger the utility value of the uncertain outcome, the more desirable is that outcome. Referring back to Figure 1 (and assuming that more than one P_j has a positive probability of occurring), it is convenient to view the consequences of each strategy as a lottery or a gamble. If the probabilities satisfy the conditions of

$$0 \leq P_j \leq 1 \text{ for } j = 1 \text{ to } m, \text{ and } \sum_{j=1}^m P_j = 1, \quad (2-2)$$

then the full set of lotteries can be mapped into utility space where the utility values are denominated in real numbers.

Certain conditions must hold before utility values can represent the relative desirability of different lotteries (132, p. 9). From Figure 1, let A_i refer to the i th lottery and $U(A_i)$ refer to the utility value of lottery i . Then the first condition which must be satisfied is the complete ordering of the lotteries. For all pairs of lotteries from the total set of lotteries, A_i must be preferred to A_j ($i \neq j$), or A_j must be preferred to A_i , or A_i and A_j must be of equal desirability. Only one of these three possibilities can be true for any pair of lotteries. The second condition which must hold true is that the ordering of lotteries must be transitive. That is, if A_1 is preferred to A_2 and A_2 is preferred to A_3 , then it must also be true that A_1 is preferred to A_3 .

If the conditions of a complete and transitive ordering of lotteries hold, then utility values denominated in real numbers can represent the relative desirability of the lotteries. It follows that the best action to pursue is the one which results in the largest (or maximum) utility.

Although this is a very general criterion of choice, the question still unanswered is how to join the possible consequences of any lottery into a single value. The sections which follow on expected utility maximization, mean-variance, and stochastic dominance are all subsets of utility maximization and describe how the consequences can be aggregated into a single utility measure.

Expected utility maximization criterion

The maximization of expected utility was first proposed by Bernoulli in 1732, proved by Ramsey in 1931 (104), and independently proved again by Von Neumann and Morgenstern in 1947 (131).¹

It was Von Neumann and Morgenstern's published work which introduced the expected utility concept to decision theory. The maximum expected utility criterion is a rule which combines the utility values of a lottery's uncertain consequences with its associated probabilities into a single utility value for that lottery. As shown in Figure 2, each consequence from Figure 1 is measured in terms of its utility $U(C_{ij})$, which is the utility of the consequence of the i th lottery if the j th outcome state prevails. P_{ij} is the probability of occurrence associated with consequence C_{ij} .²

The expected utility of any lottery L_i is

$$EU(L_i) = p_{i1} \cdot U(C_{i1}) + p_{i2} \cdot U(C_{i2}) + \dots + p_{im} \cdot U(C_{im}) \quad (2-3)$$

¹This section draws upon the material from Horowitz (59, pp. 340-350) and Luce and Raiffa (82, pp. 23-28). A list of sources providing more formal proofs is given by Borch (16, p. 33).

²The probabilities now have two subscripts indicating that the probability of any consequence occurring depends not only upon which state of nature prevails, but also upon the course of action chosen.

		Outcome States				
		S_1	S_2	.	.	S_m
Lotteries	L_1	$U(C_{11})$	$U(C_{12})$.	.	$U(C_{1m})$
	L_2	$U(C_{21})$	$U(C_{22})$.	.	$U(C_{2m})$

	L_n	$U(C_{n1})$	$U(C_{n2})$.	.	$U(C_{nm})$

		Probabilities				
P_{11}	P_{21}	.	.	P_{n1}		
P_{12}	P_{22}	.	.	P_{n2}		
.		
.		
P_{1m}	P_{2m}	.	.	P_{nm}		

Figure 2. A lottery table measured in utilities

where the expected utility is the sum of the i th row of the utility matrix times the j th column of the probability matrix of Figure 2. The lottery which results in the maximum expected utility is the lottery which is most desirable.

The maximization of expected utility is applicable to any decision making situation under risk provided the decision maker is willing to adhere to certain utility axioms. These axioms are often referred to as the Von Neumann-Morgenstern axioms of rational behavior.

The first and second axioms of rational behavior are the complete and transitive ordering of all lotteries which were described in the previous section on utility maximization. In addition to these two axioms, the following axioms must also hold.

The third axiom is continuity. Given three lotteries designated as L_1 , L_2 , and L_3 where L_1 is preferred to L_2 , and L_2 is preferred to L_3 , there then exists some probability p (where $0 < p < 1$) such that the decision maker is indifferent to choosing between L_2 and a combination of L_1 and L_3 . Mathematically, the axiom of continuity holds if the following is true for all subsets of lotteries:

$$L_2 = p \cdot L_1 + (1 - p) \cdot L_3. \quad (2-4)$$

The fourth axiom deals with the reduction of compound lotteries into simple lotteries. This means that a compound lottery can be restated as a simple lottery which is at least as preferred as the compound lottery. The right hand side of Equation 2-4 can be viewed as a compound lottery

since L_1 and L_3 are themselves composed of a series of consequences, C_{ij} 's, and associated probabilities, p_{ij} 's. The left-hand side of Equation 2-4 is the simple lottery which is at least as preferred as the compound lottery. If Equation 2-4 holds for all combinations of lotteries, the third and fourth axioms of rational behavior are satisfied.

The fifth axiom is independence. If the decision maker is indifferent to the choice between two lotteries denoted by L_1 and L_2 and is also indifferent to the choice between two other lotteries denoted by L_3 and L_4 , then a lottery, L_5 , can be constructed from lotteries L_1 and L_3 and another lottery, L_6 , can be constructed from lotteries L_2 and L_4 as

$$L_5 = p \cdot L_1 + (1 - p) \cdot L_3 \text{ and} \quad (2-5)$$

$$L_6 = p \cdot L_2 + (1 - p) \cdot L_4. \quad (2-6)$$

If the probability p appearing in Equations 2-5 and 2-6 is the same, then the decision maker will be indifferent to the choice between lotteries L_5 and L_6 .

The last axiom of rational behavior addresses the preference of lotteries constructed with unequal probabilities. If the decision maker prefers lottery L_1 to lottery L_2 and constructs two lotteries, L_3 and L_4 , by combining L_1 with L_2 , such as

$$L_3 = p \cdot L_1 + (1 - p) \cdot L_2 \text{ and} \quad (2-7)$$

$$L_4 = r \cdot L_1 + (1 - r) \cdot L_2, \quad (2-8)$$

lottery L_3 will be preferred only to L_4 if p is greater than r . If p is less than (or equal to) r , then L_4 must be preferred (or indifferent to) to L_3 .

If these six axioms of rational behavior are true for a decision maker, it is always possible to map the decision maker's preferences into utilities (82, p. 29).

If the utility mapping is denominated in monetary terms, then it is possible to restate the preference ordering of all lotteries in terms of the most preferred and least preferred lotteries. The last axiom concerning unequal probabilities can then be used to evaluate the riskiness of any lottery. This technique is often used in empirical work to construct a decision maker's preferences.¹

Alternatively, it is frequently assumed that the mapping of preferences into utility numbers can be expressed as a mathematical function. If this function is continuous and differentiable, then Equation 2-9 below can be solved via calculus to find the maximum expected utility for a decision problem involving risk. The expected utility of some risky prospect Y is

¹Refer to Binswanger (12, pp. 395-407) and Dillon and Scandizzo (28, pp. 425-435) for empirical applications of this approach.

$$EU(Y) = \int_a^b U(y)f(y)dy, a < y < b, \quad (2-9)$$

where

$U(y)$ is the mathematical form of the utility function
(expressed here in its most general form), and

$f(y)$ is the associated probability density function.¹

Although it has been shown to be theoretically possible to construct utility functions for an individual or a group of individuals (3, pp. 69-100), the problem still remains as to how a utility function or an individual's attitude toward risk (to be discussed shortly) can be measured empirically. In 1957, Luce and Raiffa (82, pp. 34-35) were somewhat pessimistic about the possibility of actually producing such a function. Luce and Suppes (83) reviewed early experimental work on measurements of attitudes toward risk. Officer and Halter (98) used approaches based on utility theory and the elicitation of certainty equivalents using hypothetical choice techniques. In 1980, Binswanger (12, pp. 395-407) used an experimental technique with actual payouts with a certain degree of success.²

At present, the elicitation of utility functions is a difficult, time consuming, and expensive exercise prone to errors and biases.

¹If the mapping is discrete, then Equation 2-3 provides the appropriate result.

²Hans Binswanger's conclusions would indicate virtually all individuals are moderately risk averse and that wealth tends to reduce risk aversion slightly, although the effect was not statistically significant.

However, much progress has been made since the mid-1950s and undoubtedly will continue to be made in the future. The weight of the empirical evidence supports the contention that the elicitation of utility functions is indeed possible. Therefore, the maximization of expected utility as the choice criterion is viable and empirically implementable.

In empirical applications, Equation 2-9 requires an exact specification of the functional form of utility. Even if an exact specification can be determined, the results are applicable only to the particular decision maker (and others, if any, who might have the identical specification of utility). Although these results would be very useful to the individual, they would probably be of little use to anyone else.

Generally, it is more useful in economic research to generate results which will be applicable to a group of decision makers instead of a single individual. This can be accomplished by placing restrictions upon the shape of the utility function. The results of the study are then applicable to all decision makers whose utility functions satisfy the stipulated restrictions. The more exacting the restrictions, the smaller the group of decision makers to which the results are applicable. There is, therefore, a trade-off between the generality to whom the results pertain and the definitiveness of the results.

Mean-variance analysis and stochastic dominance are two subsets of expected utility maximization which can be used as criteria of choice without the need to specify the exact functional form of the utility function. But before these are discussed, it is necessary to discuss a decision maker's possible attitudes toward risk.

Attitudes Towards Risk

To facilitate decision making under uncertainty, it is necessary to assume that utility can be completely described in monetary terms (such as income, wealth or money payoffs). Furthermore, it is necessary to assume that utility is nonnegative and monotonically increasing. This implies, as shown in Figure 3, that utility increases with increasing wealth, no matter what the decision maker's attitude toward risk. The three attitudes or behaviors towards risk are defined as risk aversion, risk neutrality, and risk seeking.

Risk averse behavior means an individual prefers a known situation to a risky situation even when the two situations have the same expected outcome. Utility functions of risk averse individuals are concave in wealth (i.e., they evidence decreasing marginal utility), as shown in Figures 3 and 4.¹ That is, the first derivative is positive and the second derivative is negative.

¹Formally, for a gamble with two possible outcomes, x and y , with probabilities $P(X=x)=\alpha$ and $P(X=y)=1-\alpha$, $0 \leq \alpha \leq 1$, risk aversion requires that

$$\alpha \cdot U(x) + (1 - \alpha) U(y) \leq U[\alpha \cdot x + (1 - \alpha) \cdot y].$$

Any utility function satisfying this condition for all combinations of x and y on some bounded interval I is said to be concave on I . If, in addition, the inequality is strict whenever $x \neq y$, the function is said to be strictly concave.

Strict concavity is equivalent to a positive first derivative of utility which is strictly decreasing with increasing wealth, or a negative second derivative. For a proof of this equivalence, refer to Hardy, Littlewood and Polya (50); Mangasarian (85) or Zangwill (136).

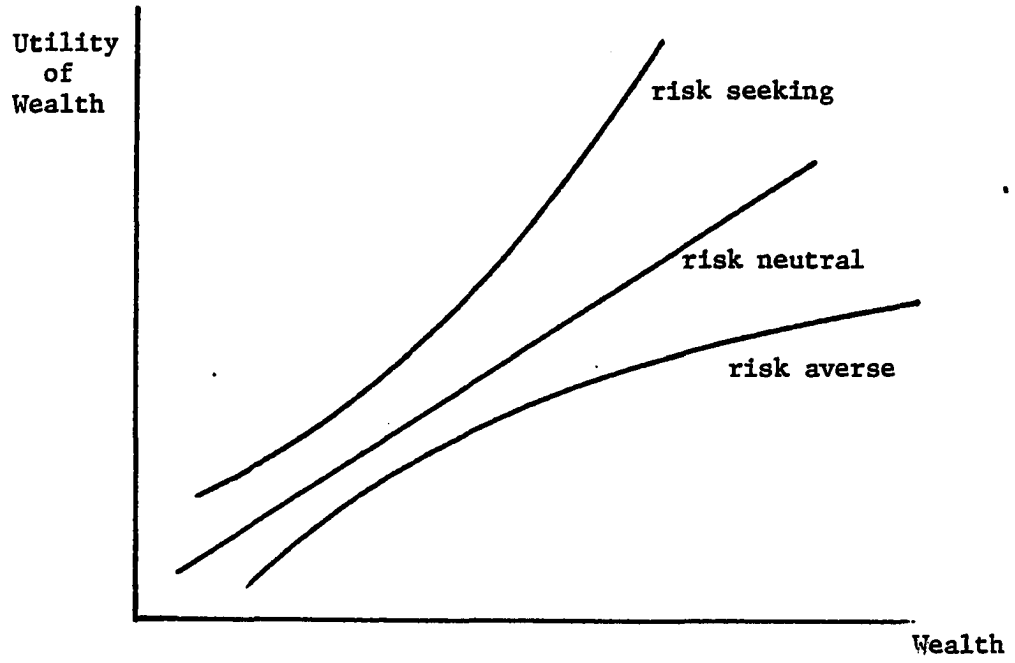


Figure 3. Behaviors toward risk

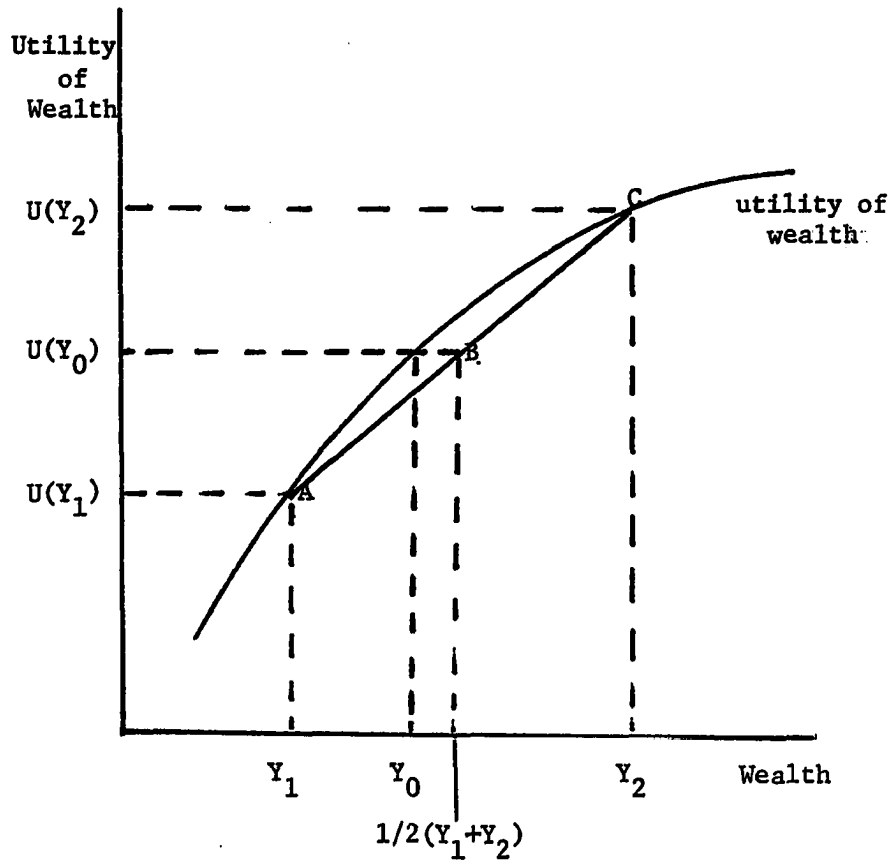


Figure 4. Risk averse behavior

Figure 4 is an example of risk averse behavior where increasing utility is measured along the vertical axis and the monetary terms are measured along the horizontal axis. In Figure 4, outcomes Y_1 and Y_2 have utility values $U(Y_1)$ and $U(Y_2)$ respectively. The linear line segment \overline{AC} describes a risky lottery composed of a combination of Y_1 and Y_2 . Point B, the midpoint of \overline{AC} , represents a risky prospect with a fifty percent probability of outcome Y_1 occurring and a fifty percent chance of Y_2 occurring. The expected value of this risky prospect is $1/2(Y_1 + Y_2)$ and the expected utility is $U(Y_0)$. A risk averse individual would be indifferent to the certain outcome Y_0 and a risky prospect of $1/2$ of Y_1 and $1/2$ of Y_2 since both result in the same level of utility. Y_0 is said to be the certainty equivalent of the risky prospect $1/2(Y_1 + Y_2)$.

Conversely, the difference between $1/2(Y_1 + Y_2)$ and Y_0 is the risk premium the individual demands to take on the risk. There are two equivalent ways of viewing the risk premium. One view is that the risk premium is the additional compensation demanded by the decision maker to take on the risk. The other view is that the risk premium is the amount of income or wealth the individual would be willing to forgo to avoid the risk. The risk premiums demanded by risk averse individuals are always positive.

A decision maker who is risk neutral is indifferent to risk and ranks prospects based upon their expected values. The utility function for such an individual is linear with a positive first derivative and a

second derivative equal to zero. The risk premium of a risk neutral decision maker is always zero.

A risk seeking individual prefers a risky prospect to a certain prospect if the two prospects have the same expected value. The utility function of a risk seeker is convex in wealth, as shown in Figure 3. The risk seeker's second derivative is positive and his risk premium is always negative. That is, the risk seeker is always willing to forgo some monetary return so as to enjoy the opportunity to take a risk.

Most studies of the firm under uncertainty assume that the decision maker is risk averse.¹ For such firms, an additional characteristic of decreasing absolute risk aversion is also often assumed. Pratt (101, p. 125) defined decreasing absolute risk aversion as

$$R_A(Y) = \frac{U''(Y)}{U'(Y)} \quad (2-10)$$

where

$U'(Y)$ is the first derivative of utility with respect to wealth, and

$U''(Y)$ is the second derivative.

Decreasing absolute risk aversion says the decision maker is more willing to accept a given level of risk, the higher is the decision maker's income. For a risk averter, absolute risk aversion is positive,

¹Friedman and Savage (45, pp. 293-297) argue convincingly that utility functions must evidence both risk averse and risk seeking sections in order to portray the totality of human behavior. However, most studies typically restrict their analysis to the concave section of the utility curve.

and decreasing absolute risk aversion implies that the third derivative of utility is positive (4, p. 122).¹

The maximization of expected utility as the choice criterion in conjunction with a positive first derivative and negative second derivative of utility are sufficient to develop a fairly extensive theory of the firm in a world of uncertainty. As Samuelson suggests (109, p. 537), most of the important aspects of risk theory can be derived from these general assumptions. Sandmo (110, pp. 65-73) provides an excellent discussion of techniques as well as some comparative static results for a competitive firm operating in a risky environment. Horowitz (59, pp. 363-415) also presents a fairly detailed discussion of a number of models in which risk has been included.

However, the use of expected utility as the appropriate choice criterion is not universally accepted. Allias (Borch, 16, pp. 62-63 and Markowitz, 87, pp. 220-221) developed an example which showed that individuals do not act rationally. People were inconsistent in their choices and therefore the axioms upon which expected utility is predicated are violated and expected utility is not a valid choice criterion. A heated

¹A second measure of the degree of risk aversion is Pratt's relative risk aversion which is:

$$R_R(Y) = - \frac{U''(Y)}{U'(Y)} \cdot Y = R_A(Y) \cdot Y.$$

This measure expresses the willingness of an individual to accept risk when the size of the risk and income are increased proportionately. Arrow (5, p. 111) argues that relative risk aversion should increase as income increases; however, this idea is not universally accepted (88, p. 410) and will not be pursued further here.

debate ensued during the 1950s over the validity of expected utility maximization. Although the issue is moot, most economists today accept rational behavior as a reasonable assumption on the grounds that there is no better tenet of human behavior available.

As discussed earlier, it is generally more useful in economic research to define some characteristics and restrictions which a group of utility functions must satisfy than to know the exact specification of one particular utility function. In this manner, the results can be applied to all individuals whose utility functions satisfy the stated conditions even though the exact specifications of the utility functions will undoubtedly not be the same. Furthermore, the more general the restrictions, the broader will be the applicability of the results. Within this context several empirical approaches have been used to model firm behavior under uncertainty. Two of the most powerful methods will briefly be reviewed.

Mean-Variance Analysis

The major tenet of mean-variance analysis is that a utility function can be completely described in terms of its mean and variance. That is, expected utility is composed of the first statistical moment about the origin and the second moment about the mean. All higher order statistical moments about the mean do not matter, because they do not exist (quadratic utility function), can be safely ignored through a Taylor series expansion, or can be restated in terms of the variance. Johnson (66) contains an excellent discussion of the applicability of

mean-variance analysis. See also Farrar (33, pp. 20-22) and Kaplan (68, p. 426).

The premise upon which mean-variance analysis is based is that the mean of the distribution represents the expected return and that the variance (or standard deviation, or some other statistical moment measuring variation) is a proxy for the riskiness of the prospect.

Markowitz (86) first promoted the use of mean-variance preference ordering in the analysis of risk. In this theory, the individual is assumed to make decisions by evaluating the trade-offs between return and variance of the risky prospects. Denoting the distributions of two alternative risky prospects as F and G, a risk averse decision maker will prefer distribution F over distribution G if

$$E(X_F) \geq E(X_G) \text{ and} \quad (2-11)$$

$$V(X_F) \leq V(X_G) \quad (2-12)$$

where

E is the mean return and

V is the variance

and at least one of the inequalities is strict. In this sense, F is said to dominate G in expected return-variance preference ordering and G can be eliminated from further consideration as a noncontender with no loss of optimality. This is to say that G is inefficient and will never be chosen by a risk averse decision maker.

If only one of the inequalities in (2-11) and (2-12) is true, then neither dominates the other and both F and G would remain in the efficient set. In this situation, the choice depends on the individual's personal preferences concerning the trade-off between return and risk. More must be known about the specification of the individual's utility function before a choice can be made.

Although the early work on expected return-variance analysis was done in terms of expected values of gain, the theory has been reformulated by Arrow (4), Samuelson (108), and others to be consistent with the axioms of expected utility maximization.

Beginning with the early work done by Freund (43), Markowitz (86, 87) and Tobin (121) through Sharpe (113), the primary method of solution is quadratic programming. But the use of quadratic programming as a realistic procedure to solve expected utility problems is subject to criticism on two points. First, the use assumes that the underlying distribution is symmetrical, but in many applications in agriculture, this assumption is unrealistic (123, p. 355; 15, p. 288 and 51, pp. 490-491).¹ Second, the quadratic utility functions evidence increasing absolute risk aversion but this contradicts what many economists believe people's behavior to be (5, pp. 96-97 and 34, p. 6).

¹Numerous authors maintain (for example, 34, pp. 6-8 and 122, p. 13) the distribution must be normal which is more restrictive than mere symmetry. But as Johnson (66) has shown, mere symmetry is sufficient.

Regardless of the criticisms directed at mean-variance analysis, many useful inferences about economic behavior have been generated from its use.

Stochastic Dominance

Partly due to the two criticisms mentioned above, many researchers have pursued stochastic dominance as an alternative to quadratic programming (132, pp. 34-35; 92 and 93). A detailed development of stochastic dominance will be delayed until the next chapter, but the basic concept will be introduced here.

In a fashion quite similar in purpose to quadratic programming, the full set of possible lotteries is reduced to an efficient set by eliminating inefficient prospects from further consideration. Quadratic programming generates an efficient set by maximizing the mean for a given variance or minimizing the variance for a given mean. Stochastic dominance generates an efficient set by placing sign restrictions on the successive derivatives for the class of utility functions under consideration. For example, first degree stochastic dominance stipulates that the first derivative of utility must be positive; second degree stochastic dominance stipulates the second derivative must be negative; and the nth degree stochastic dominance places a sign convention upon the nth derivative of utility. Ultimately, a sufficiently high degree of dominance can be reached where there is only one efficient prospect remaining in the efficient set (or more than one, if all are of equal desirability).

Expected Utility Measured in Terminal Wealth

Mean-variance analysis and stochastic dominance are both methods of measuring expected utility. The question still remains as to what utility should measure. It is generally accepted by economists that the most desirable measure is consumption. That is, the most appropriate objective function is the maximization of the expected utility of lifetime consumption (Hirshleifer, 57 and Hey, 56, pp. 70-82.) In general, the argument is that the decision maker should choose $C_1, C_2, \dots, C_T, C_{T+1}$ such that $U(C_1, C_2, \dots, C_{T+1})$ is maximized subject to $W_t \geq C_t \geq 0$ ($t=1$ to $T+1$), where W_t is defined recursively by $W_{t+1} = W_t(1+r) + Y_t - C_t$ ($t=1$ to $T+1$). W_t is the net wealth in year t ; C_t is consumption, Y_t is income; and r is the rate of interest between periods. There are assumed to be T periods in the individual's expected life; therefore, C_{T+1} can be viewed as the individual's at death bequests. Those unfamiliar with the solution of this model are referred to Henderson and Quandt (54, pp. 297-309) under conditions of certainty; Sandmo (111) considers separately both income uncertainty and uncertainty about interest rates; Dreze and Modigliani (30) consider income uncertainty and rate of return uncertainty; and Levhari and Srinivasan (77) address the solution of infinite horizon models.

Assuming that the utility function is additively separable, then the maximization of $U(C_1, C_2, \dots, C_{T+1})$ can be rewritten as:

$$U(C_1, C_2, \dots, C_{T+1}) = \sum_{t=1}^{T+1} \beta^t u(C_t) \quad (2-13)$$

where

β^t (t=1 to T+1) is the vector of weights attached to consumption.

There is empirical evidence which suggests that a relationship exists between income and consumption in any period (21, p. 172 and 18). For expository purposes, this relationship is

$$C_t = dY_t; \quad 0 < d < 1. \quad (2-14)$$

The utility maximization problem (2-13) can be rewritten using (2-14) as

$$U = \sum_{t=1}^{T+1} \beta^t u(dY_t) \quad (2-15)$$

Since consumption plus savings must equal income, and using (2-14), savings is

$$S_t = (1-d)Y_t; \quad 0 < d < 1, \quad (2-16)$$

and therefore $Y_t = \left(\frac{1}{1-d}\right)S_t$. Substituting this expression for income into (2-15),

$$U = \sum_{t=1}^{T+1} \beta^t u\left[\left(\frac{d}{1-d}\right)S_t\right] \quad (2-17)$$

Since the vector of weights on consumption β^t and d are constant, both can be subtracted from the utility function without altering utility rankings (54, p. 22). By defining a new vector of weights $\alpha^t = \beta^t \left(\frac{d}{1-d}\right)$,

(2-17) can be rewritten as

$$U = \sum_{t=1}^{T+1} \alpha^t u(S_t) \quad (2-18)$$

Therefore, utility can be maximized by finding the optimum set of consumption weights or, equivalently, the optimum set of savings rates.

For any period, savings are used to invest in new capital goods or

$$S_t = \sum_{i=1}^k P_{it} I_{it}, \quad (2-19)$$

where

P_{it} is the price of capital good i ($i=1$ to k), and

I_{it} is the investment in good i in time t .

Total investment in any one capital good throughout the horizon is

$$I_i = \sum_{t=1}^{T+1} I_{it} + K_{i0}, \quad (2-20)$$

where K_{i0} is the initial endowment of capital good i . A weighted average price for each capital good can be calculated by dividing the value of all purchases by the total investment and adjusting for time preferences or

$$P_i = \left(\sum_{t=1}^{T+1} P_{it} \alpha^t I_{it} + K_{i0} \right) / I_i. \quad (2-21)$$

where P_i is the weighted price. Then maximizing $\sum_{i=1}^k P_i I_i$ is the same

as maximizing $\sum_{i=1}^k \sum_{t=1}^{T+1} P_{it} \alpha^t I_{it} + K_{i0}$ which by (2-19) is the same as maximizing (2-18). The problem now has been restated as one of maximizing ending asset values at some particular price. Boussard (18, pp. 469-471) argues that use of the "turnpike theorem" allows considerable flexibility in assigning terminal prices to assets. With a sufficiently long horizon, the expansion path of the firm is determined more by technical coefficients than prices (within a reasonable range) and therefore firms with differing initial endowments and facing differing price vectors will still have highly similar solutions. Hence, it is justifiable to maximize net terminal wealth (ending asset values less outstanding debt), in place of the original utility of consumption problem of (2-13).

Equivalently, Lutz and Lutz (84, p. 17) suggest an entrepreneur will want to maximize the rate of return on owned capital, or maximize ending wealth, or net worth. Furthermore, as Fama (32) has shown, the properties of nonsatiation and risk aversion in the utility specification for multiperiod consumption are identical to a utility specification in net terminal wealth. Therefore, the appropriate objective function with which to incorporate risk into the decision making process is the maximization of expected utility of net terminal wealth.

The next chapter presents the theory of stochastic dominance. Stochastic dominance theorems are used to choose the risky prospect which produces the maximum expected utility from a set of possible risky prospects.

CHAPTER III. STOCHASTIC DOMINANCE

For the decision maker who must choose between two risky prospects and knows some general but limited information about his utility function, the existence or nonexistence of a stochastic dominance relationship between the prospects is purely a mathematical question.¹

A risky prospect is any random variable whose values occur by chance. It is not important whether the probabilities are "objective" or "subjective" in nature. However, it is assumed the probabilities obey the axioms of mathematical probability theory. Probability theory will not be developed here; readers interested in its foundations and concepts should see such works as Feller (36 and 37), Lowe (81), Brieman (22), Fishburn (41) and Larson (73).

Stochastic Dominance Theorems

The theorems of stochastic dominance have the common theme of providing a basis for choosing between two risky prospects. All individuals whose utility functions conform to the theorems will regard one prospect as being at least as desirable as (or more desirable than) the other. This process of comparing two prospects is accomplished through degrees. First degree stochastic dominance is the least restrictive and provides the most general results. Second degree stochastic dominance is

¹This chapter draws heavily upon the works of Fishburn and Vickson (42, pp. 39-113), Anderson and Dillon (3, pp. 281-317) and Meyer (92, pp. 326-336 and 93).

more restrictive than first degree and its power to differentiate between two risky prospects is greater. However, a second degree stochastic dominance efficient set is applicable to a smaller set of decision makers than is first degree. Similarly, successively higher degrees of stochastic dominance can be employed.

Defining the term $>_i$ as "strictly preferred in the sense of ith degree stochastic dominance," the stochastic dominance theorems can be defined in general as

$$F >_i G \text{ if and only if } EU(X)_F > EU(X)_G \quad (3-1)$$

for all $u \in U_i$,

where

F is the cumulative distribution function of some continuous risky prospect A ,¹

G is the cumulative distribution function of some other continuous prospect B ,

$EU(X)_F$ is the expected utility of prospect A ,

$EU(X)_G$ is the expected utility of prospect B , and

¹Consistent with conventional notation, upper case letters (X, Y) will be used to denote random variables and the corresponding lower case (x, y) to denote a particular value of the random variable. Similarly, upper case letters (F, G) will be used to denote cumulative density functions and the lower case (f, g) to denote the respective probability density function. For a continuous variable, the cumulative density function and the probability density function are related by the relationship $F(x) = \int^x f(y)dy$ and $f(x) \geq 0$ for all x .

$u \in U_i$ is the subset of all utility functions satisfying the restrictions imposed by degree i .¹

The restrictions imposed on the form of the utility functions deal with strict positivity or negativity restrictions on the derivatives of utility with respect to the uncertain variable.

First degree stochastic dominance (FSD)

Nonsatiation is the most general restriction commonly imposed on utility of wealth $U(X)$. Nonsatiation is mathematically expressed as $U(X)$ must be nondecreasing in X over some interval I , or $U(x) < U(y)$ if $x < y$, for all $x, y \in I$. Equivalently, utility is said to be monotonically increasing with respect to wealth. For convenience, it is assumed that utility is continuous and once differentiable. The subclass of all utility functions which satisfy the restriction of nonsatiation is defined as

$$U_1 = \{u: u, \frac{dU}{dX} \text{ is continuous and bounded on } I, \quad (3-2)$$

$$\frac{dU}{dX} > 0 \text{ on } I^o, \}$$

where

$\frac{dU}{dX}$ is the first derivative of utility with respect to wealth.

¹It is also assumed here that the utility functions are well-defined and finite over the entire range of possible random outcomes. Furthermore, the random variables are bounded from below on a closed interval (42, p. 51).

I denotes a closed interval such as $[0, \infty]$ or $[a, b]$, and I° is the interior of I or $(0, \infty)$ or (a, b) .¹

Aside from nonsatiation, the class of utility functions U_1 defined in (3-2) says nothing about attitudes toward risk. Both risk averse and risk seeking behaviors are consistent with this class. The only restriction imposed by first degree stochastic dominance is that more wealth is preferred to less.

First degree stochastic dominance will result in the unanimous preference of one risky prospect over another for all decision makers whose utility functions belong to U_1 as defined in (3-2) if, for every value of x belonging to I , the cumulative density function of the preferred prospect is less than the cumulative density function of the other prospect. Equivalently, first degree stochastic dominance can be stated as

$$F \succ_1 G \text{ if and only if } D^1(x) \geq 0 \text{ for all } x \in I, \text{ and} \\ D^1(x) > 0 \text{ for at least one value of } x, \quad (3-3)$$

where

F, G denote the cumulative distributions of two risky prospects defined on the interval I , and

$$D^1(x) = G(x) - F(x) \text{ for all } x \in I.$$

In (3-3), if $D^1(x) = 0$ for all values of x , then neither F dominates G nor G dominates F . In this case, the decision maker is

¹The first derivative may equal zero or be undefined at the endpoints of I without affecting the analysis.

equally indifferent to F and G, and both will remain in the efficient set. Conversely, if $D^1(x) < 0$ for all values of x , then G dominates F.

As a simple example of first degree stochastic dominance, assume the decision maker is faced with the two alternative courses of action, A and B, shown in Figure 5, where F is the cumulative density function of A and G is the cumulative density function of B. Clearly distribution F is preferred to distribution G since it lies farther to the right and therefore has a higher expected outcome for every value of x . Conversely, $D^1(x)$ is greater than zero for every value of x , and according to (3-3), F is said to dominate G in the sense of first degree stochastic dominance. In the simple example portrayed in Figure 5, all decision makers satisfying U_1 will prefer F to G and will select strategy A as the preferred course of action. Strategy B is obviously an inefficient course of action and can be ignored in subsequent analysis.

However, Figure 6 is an example where first degree stochastic dominance can not choose between two risky prospects because the inequality of (3-3) does not hold for all values of x . The quantity $D^1(x)$ is negative for values between a and b , zero at b , and positive for values of x greater than b . In this case, neither distribution is dominant in the sense of first degree stochastic dominance. Both F and G will remain in the first degree efficient set. It remains for the decision maker to choose between the two distributions based upon his personal attitude toward risk.

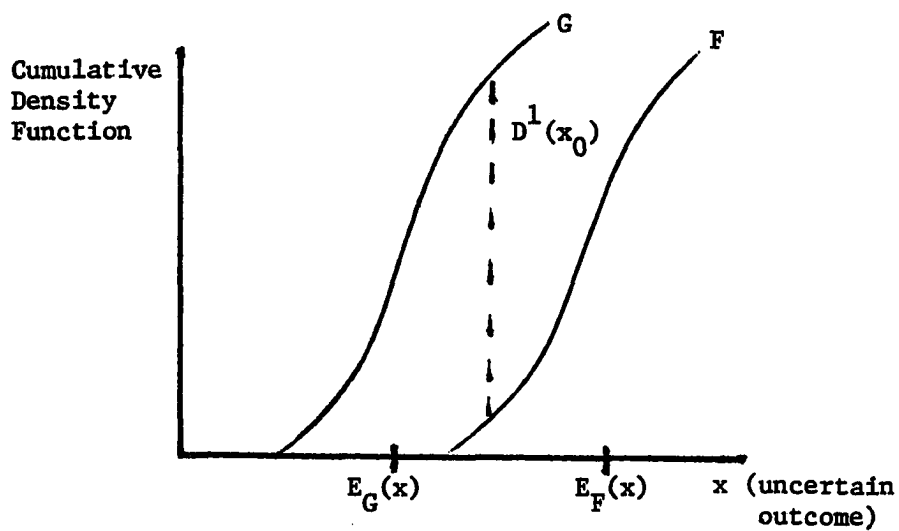


Figure 5. First degree stochastic dominance (FSD) where F dominates G

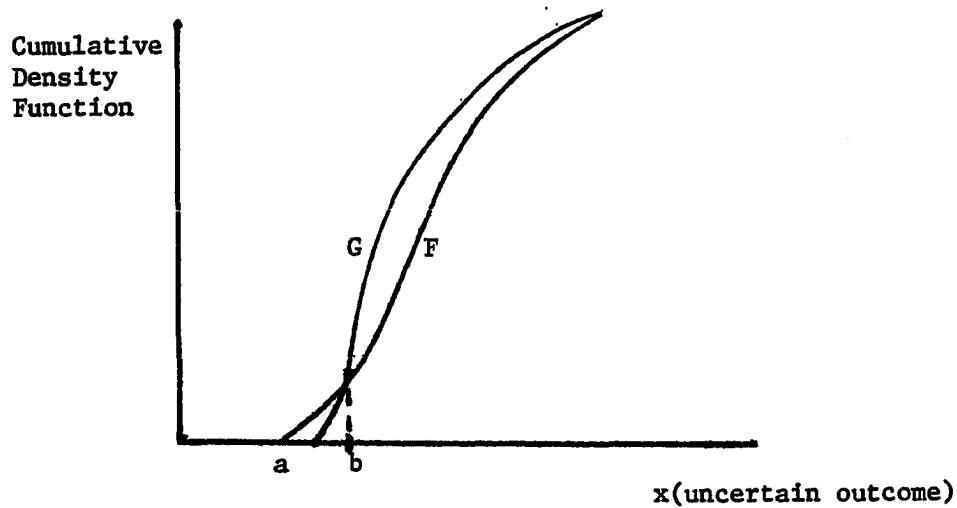


Figure 6. FSD where F and G cross

Second degree stochastic dominance (SSD)

For a decision maker to choose between two distributions such as those portrayed in Figure 6, an additional restriction must be imposed upon the subclass of utility functions belonging to U_1 . This restriction concerns the decision maker's attitude toward risk. As explained in the previous chapter, risk aversion is a commonly accepted trait of individuals who behave rationally. As discussed at that time, risk averse utility functions must be increasing and concave in wealth. Equivalently, the first derivative of utility with respect to wealth must be positive and the second derivative must be negative. The class of utility functions for all risk averse decision makers is

$$U_2 = \{u: u \in U_1, \frac{d^2U}{dx^2} \text{ is continuous and bounded on } I, \\ \frac{d^2U}{dx^2} < 0 \text{ on } I^o\} . \quad (3-4)$$

Condition (3-4) says that the class of risk averse utility functions, U_2 , is a subset of U_1 defined in (3-2) that also possesses negative second derivatives with respect to wealth.

Define $F^2(x)$ as the area under the graph of the cumulative density function of $F(y)$ from $y=0$ to $y=x$ or

$$F^2(x) = \int_0^x F(y)dy, \quad x \in I. \quad (3-5)$$

Similarly define the area under G as

$$G^2(x) = \int_0^x G(y) dy, \quad x \in I. \quad (3-6)$$

And define the difference between G^2 and F^2 as

$$D^2(x) = G^2(x) - F^2(x). \quad (3-7)$$

Then distribution F is said to dominate distribution G in the sense of second degree stochastic dominance or

$$\begin{aligned} F >_2 G \text{ if and only if } D^2(x) \geq 0 \text{ for all } x \in I \text{ and} \\ D^2(x) > 0 \text{ for at least one value of } x. \end{aligned} \quad (3-8)$$

If $D^2(x) \leq 0$ for all values of x and the strict inequality holds for at least one value of x , then distribution G is said to dominate distribution F in the sense of second degree stochastic dominance. If $D^2(x) < 0$ for some values of x and $D^2(x) \geq 0$ for other values of x , then F and G are of equal desirability in the sense of second degree stochastic dominance.

Intuitively, a positive difference in (3-8) implies that F lies more to the right than G in terms of differences in the areas between the cumulative density functions. In Figure 7, distribution G starts out smaller than F but catches up and crosses F as x becomes larger. However, first degree stochastic dominance can not choose between F and G, i.e., neither $F >_1 G$ nor $G >_1 F$ is true. Since the difference in areas between F and G is positive in Figure 7 (area A is clearly larger

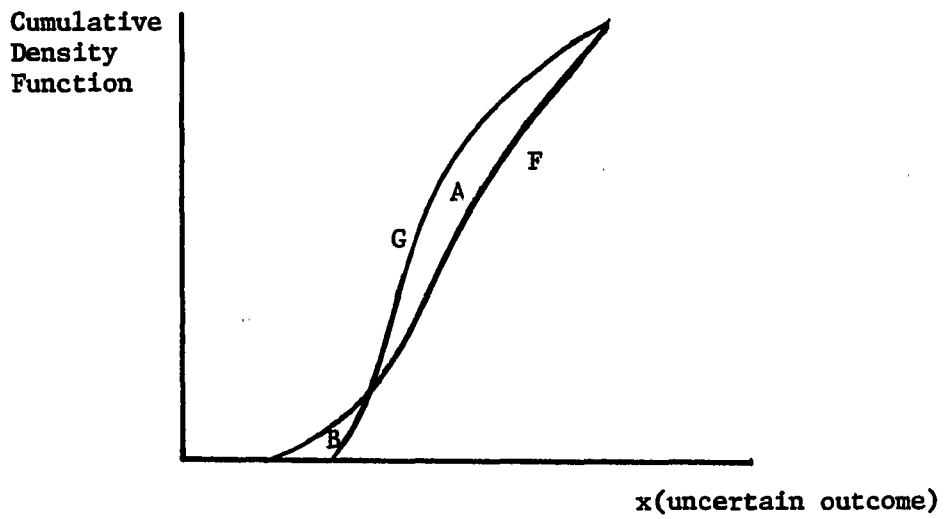


Figure 7. FSD where F and G cross but F dominates G under second degree stochastic dominance

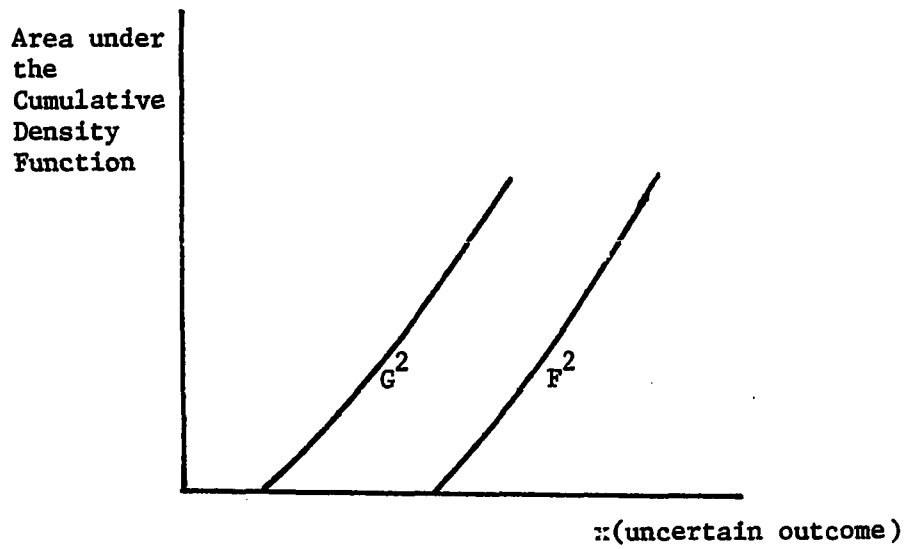


Figure 8. Second degree stochastic dominance (SSD) where F dominates G

than area B), F lies farther to the right than G, and F dominates G in terms of second degree stochastic dominance.

Alternatively, as in Figure 8, $F^2(x)$ and $G^2(x)$ are graphed instead of $F(x)$ and $G(x)$. Clearly $F^2(x)$ lies to the right and below $G^2(x)$ for all values of x (i.e., $D^2(x) > 0$ for all x) and F therefore dominates G in the sense of second degree stochastic dominance. In Figure 9, F^2 and G^2 cross; in this situation, second degree stochastic dominance can not choose the dominant prospect between F and G and therefore both would remain in the second degree efficient set.

Third degree stochastic dominance (TSD)

Nonsatiation and risk aversion are widely accepted characteristics of rational behavior. Moving from second degree stochastic dominance to third degree stochastic dominance requires an assumption about rational behavior which is not so widely accepted. The necessary assumption is that decreasing absolute risk aversion is representative of rational economic behavior. Although some economists (4, 5 and 101) advocate the inclusion of decreasing absolute risk aversion, as defined in Equation (2-10) in the decision making process, this is not universally accepted. Assuming, for the moment, that decreasing absolute risk aversion is an attitude toward risk evidenced by a certain set of risk averse decision makers, these decision makers could be defined as belonging to U_3 .

The subclass of U_3 of utility functions is defined as

$$U_3 = \{u:u \mid U_2 \frac{d^3U}{dx^3} \text{ is continuous and bounded on } I, \\ \text{and } \frac{d^3U}{dx^3} > 0 \text{ on } I^0 \}. \quad (3-9)$$

The subclass of utility functions contained in U_3 consists of those in which utility increases at a decreasing rate, the absolute value of which becomes smaller as wealth increases. This condition is satisfied if

$\frac{dU}{dX} > 0$ (from U_1), $\frac{d^2U}{dX^2} < 0$ (from U_2) and $\frac{d^3U}{dX^3} > 0$. The effect of a positive third derivative is to slow the rate at which the utility function bends back toward the horizontal.¹

Defining $F^3(x)$, $G^3(x)$ and $D^3(x)$ as

$$F^3(x) = \int_0^x F^2(y)dy, \quad (3-10)$$

$$G^3(x) = \int_0^x G^2(y)dy, \text{ and} \quad (3-11)$$

$$D^3(x) = G^3(x) - F^3(x), \quad (3-12)$$

then third degree stochastic dominance is defined as

¹A positive third derivative is a necessary condition for decreasing absolute risk aversion. It is not, however, a sufficient condition since it is possible to have $\frac{d^3U}{dX^3} > 0$ and not have decreasing absolute risk aversion. An example of such a utility function is the almost quadratic function $U(x) = x - cx^2 + ex^3$. The third derivative is positive if $e > 0$. However, if $0 < e < 4c^2/6$, then absolute risk aversion is positive and increasing for all values of x .

$F >_3 G$ if and only if,

a) $D^3(x) \geq 0$ for all $x \in I$ and

b) $E(X)_F \geq E(X)_G$

and either $D^3(x) > 0$ for at least one x or

$E(X)_F > E(X)_G$. (3-13)

If (3-13) is true, distribution F is said to dominate distribution G in the sense of third degree stochastic dominance. For all individuals whose utility functions satisfy U_3 , F is unanimously preferred over G and distribution G can be eliminated from further consideration. Figures 9 and 10 are examples of two distributions F and G where F dominates G in the sense of third degree stochastic dominance but neither $F >_2 G$ or $G >_2 F$ is true.

Nth degree stochastic dominance

First, second and third degree stochastic dominance theorems were developed by imposing sign restrictions on the corresponding derivatives of utility with respect to wealth. The results of each successively higher degree of dominance are applicable to a smaller and smaller subclass of decision makers. For degrees of stochastic dominance higher than three, the theorems of stochastic dominance have been generalized by Jean (64, p. 151). The nth subclass of decision makers whose utility functions satisfy stochastic dominance of degree n is classified as

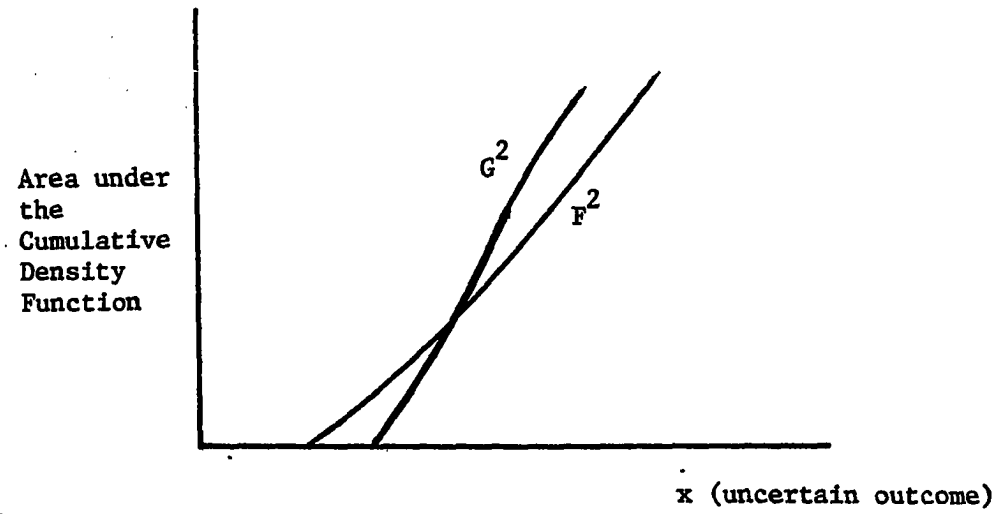


Figure 9. SSD where F and G cross

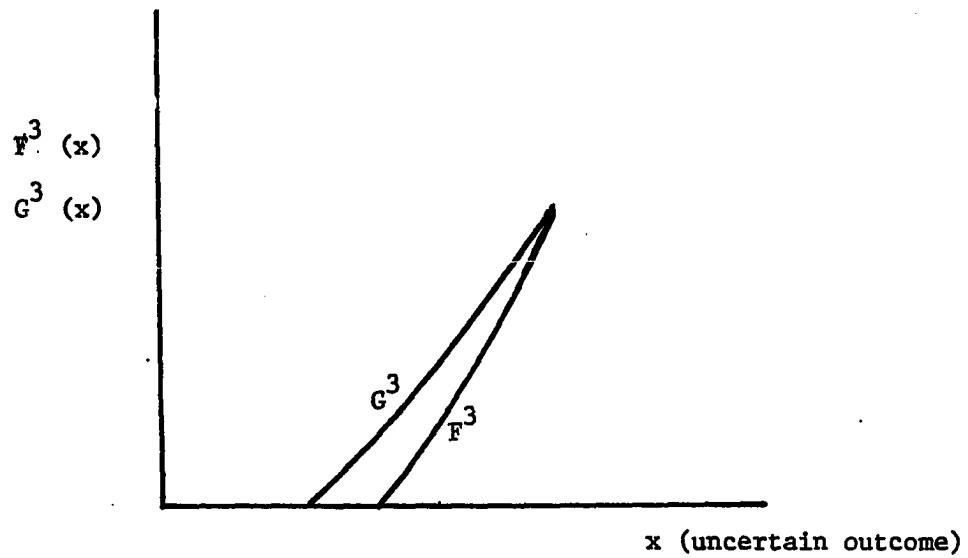


Figure 10. Third degree stochastic dominance where F dominates G

$$U_n = \left\{ u: u \in U_{n-1}, \frac{d^n U}{dX^n} \text{ is continuous and bounded on } I, \text{ and} \right.$$

either a) $\frac{d^n U}{dX^n} < 0$ on I^o if $n = 4, 6, 8, \dots$ },

or b) $\frac{d^n U}{dX^n} > 0$ on I^o if $n = 5, 7, 9, \dots$ } (3-14)

Then distribution F is preferred to distribution G in the sense of the n th degree stochastic dominance.

$$F >_n G \text{ if and only if } D^k(b) \geq 0 \text{ for } k = 1, 2, \dots, n-2$$

$$\text{and } G^{n-1}(x) \geq F^{n-1}(x) \text{ for all } x, \quad (3-15)$$

where

$$D^k(b) = G^k(b) - F^k(b),$$

$$G^k(b) = \int_0^b G^{k-1}(y) dy,$$

$$F^k(b) = \int_0^b F^{k-1}(y) dy, \text{ and}$$

b is the maximum value that x can take.

With first degree stochastic dominance, nonsatiation of wealth is assumed to be consistent with rational behavior. Second degree stochastic dominance requires that aversion to risk and nonsatiation are tenets of rational behavior. Moving to the third degree requires stepping on thinner ice in that the decision maker who behaves rationally is assumed to be nonsatiated, risk averse and decreasingly risk averse in absolute terms as wealth increases. Progressing to the fourth and higher degrees of stochastic dominance is difficult to defend in terms of

rational behavior because the fourth derivative (and higher derivatives) of utility with respect to wealth carry no economic meaning. Because of the absence of a linkage between a sign restriction on a high order derivative and its implication to rational behavior, stochastic dominance is seldom employed in economic analysis in degrees beyond the third.

The theorems presented in (3-3), (3-8), and (3-13) have appeared in equivalent forms throughout much of the literature of mathematics and mathematical statistics. Interested readers are referred to Blackwell and Girschick (13), Quirk and Saposnik (103), Fishburn (41), Hadar and Russell (48), Hanock and Levy (49), Rothschild and Stiglitz (107) and Whitmore and Findlay (133) for examples of such theorems in the economics of decision making under risk.

Special Properties of Stochastic Dominance

First, second and third degree stochastic dominance preference orderings have a number of properties that are very important in practical applications (42, p. 64). These include:

- 1) Asymmetry: if $F >_1 G$, then it is false that $G >_1 F$;
- 2) Transitivity: if $F >_1 G$ and $G >_1 H$, then $F >_1 H$ for some degree i ;

and

- 3) Implied dominance: $F >_1 G$ implies $F >_2 G$ implies $F >_3 G$.

The first property of (3-16) states that either F is preferred to G or G is preferred to F or neither is preferred in the sense of the i th degree stochastic dominance. Only one of these can be true for any pair of unequal distributions. The transitive property infers that if F is found to dominate G and G is found to dominate some other distribution H , then F and H need not be compared because it must be true that F dominates H . If a risky prospect is determined to be inefficient when compared to a given distribution in the efficient set, then it must be true that the inefficient distribution is also inefficient for all other members in the efficient set. The third property of implied dominance says that if one class of utility functions is a subset of another, then dominance in the larger class implies dominance in the smaller class. When two distributions are compared and one is determined to be preferred in the sense of first degree stochastic dominance, then it is not necessary to compare the two at second, third or higher degrees because first degree dominance implies dominance at all higher degrees. However, the converse of the third property is not true. Other special properties of interest are the following.

Equal means

If two distributions F and G have equal means, then $F >_1 G$ or $G >_1 F$ is impossible. This is obvious when considering linear utility functions. Furthermore, with equal means and $F >_2 G$, then it must be true that the variance of F is smaller than the variance of G (132, p. 78). A similar relationship holds (without the strict inequality) for

third degree stochastic dominance. That is, with equal means and $F \succ_3 G$, the variance of F is smaller than the variance of G (124, p. 78).

Discrete probability distributions

The theorems of stochastic dominance presented in (3-3), (3-8), and (3-13) were structured in terms of continuous cumulative density functions F and G . These have corollaries for cases involving discrete, finite distributions (3, pp. 282-290 and 99, pp. 119-120).

The discrete cumulative density functions of F and G are defined as

$$\begin{aligned}
 F(X) &= \sum_{\substack{\text{all} \\ x_i \leq x}} f(x_i) \text{ and} \\
 G(X) &= \sum_{\substack{\text{all} \\ x_i \leq x}} g(x_i). \tag{3-16}
 \end{aligned}$$

The quantities $F(X)$ and $G(X)$ from (3-16) can now be compared using (3-3) to test for first degree stochastic dominance.

The discrete forms of $F^2(X)$ appearing in (3-5) and $G^2(X)$ appearing in (3-6) are

$$\begin{aligned}
 F^2(x_r) &= \sum_{i=2}^r f(x_{i-1}) \Delta x_i, \text{ and} \\
 G^2(x_r) &= \sum_{i=2}^r g(x_{i-1}) \Delta x_i, \tag{3-17}
 \end{aligned}$$

where

$$\Delta x_1 = x_1 - x_{1-1}, \text{ and}$$

$$F^2(x_1) = G^2(x_1) = 0.$$

The difference between $G^2(x_r)$ and $F^2(x_r)$ is calculated as before and Theorem 3-8 is used to determine second degree stochastic dominance.

The discrete forms of $F^3(X)$ and $G^3(X)$ in (3-10) and (3-11) are

$$F^3(x_r) = \left(\frac{1}{2}\right) \sum_{i=2}^r [F^2(x_i) + F^2(x_{i-1})] \Delta x_i \text{ and}$$

$$G^3(x_r) = \left(\frac{1}{2}\right) \sum_{i=2}^r [G^2(x_i) + G^2(x_{i-1})] \Delta x_i \quad (3-18)$$

where

$$\Delta x_1 = x_1 - x_{1-1}, \text{ and}$$

$$F^3(x_1) = G^3(x_1) = 0.$$

Similarly, $D^3(X)$ is calculated as before and Theorem (3-13) is used to compare F and G for third degree stochastic dominance.

Alternative Approaches to Stochastic Dominance

The theorems of stochastic dominance theory presented thus far describe the preferences of a group of individuals by imposing sign restrictions upon the derivatives of their utility functions. Meyer maintains that this does not result in a unique specification of the decision maker's preferences. Given a functional form of utility, the signs of its derivatives are unaltered by positive linear

transformations. Because of this inconvenience, Meyer maintains it is preferable to restate utility in terms of absolute risk aversion and to place restrictions on the size this value can assume.

Meyer's approach suffers two weaknesses. The first is that some assumption must be made that decreasing, constant or increasing absolute risk aversion is representative of rational economic behavior. The second weakness is that of determining to whom the results are applicable. That is, if one prospect is shown to be preferred to another for individuals whose absolute risk aversion is between .4 and .6, who are these individuals? For a thorough development of this approach, see Meyer (92 and 93).

Another approach to the theory of stochastic dominance is convex stochastic dominance (55, p. 337). The decision theory developed thus far can address independent probability functions only on a pairwise basis. The application of dominance theory to mixtures or convex linear combinations of probability distributions is known as convex stochastic dominance. In situations where the number of pairwise combinations to be performed is prohibitively large in terms of computational expense, convex stochastic dominance can be an aid in reducing the size of the efficient set. Fortunately, it will not be needed here. See Anderson, et al., (3) and Hestenes (55) for a detailed development of convex stochastic dominance.

With the completion of the development of stochastic dominance theory as the appropriate method of maximizing expected utility of a rational decision maker, it is now necessary to turn attention to the theory of the firm.

CHAPTER IV: THE THEORY OF THE FIRM

Chapter II developed the reasons why risk should be incorporated into the decision making process. Chapter III presented a methodology for selecting the most desirable strategy from a set of risky strategies when the exact specification of the utility function is unknown. This chapter will develop the several parts of the theory of the firm. It traces the flow of funds through the firm. It identifies the sources of risk to the agricultural firm and discusses the interrelationships between the sources of risk. It reviews the fifty-year old question of whether financing matters. Last it discusses the conceptual considerations for the types of financing commonly available to the family farm.

Decision Making from a Whole Farm Perspective

The theory of the firm in agriculture encompasses the integration of the three microtheories of production, investment, and finance.¹ Production deals with the issues of what commodities should be produced in what quantities using which technologies. Investment addresses the question of finding the most efficient combination of assets to provide the necessary inputs into the production process. Finance answers the question of what is the best way to acquire the needed assets.

¹Many authors define marketing as a separate microtheory (8, pp. 5-7 and 75, p. 19). Without question, the theory of marketing is important and is given appropriate recognition in this study. It is, however, treated as a subset of production microtheory.

Each microtheory contributes a component of risk to the total enterprise risk of the decision making firm.¹ Variability in yields and prices causes production risk. Investment risk results from the ownership of assets whose economically useful lives exceed the production cycle.² Financial risk is due to fixed financing costs which result from the use of nonequity money capital (128, pp. 779-783 and 130, pp. 53-58).

The majority of the literature written under the rubric of production management theory and financial management theory (with the notable exception of Vickers) addresses only one or at most two of the microtheories. However, the three microtheories are not independent. As shown in Figure 11, the major nexus of management decision making flows from production through investment to finance. That is, the decision maker first decides what to produce and what technologies to employ. The decision maker next determines what assets are needed to provide the necessary inputs to the production process. Finally, the decision maker decides how the assets should be acquired. This sequence of events is represented by the heavy arrows in Figure 11.

¹As discussed in Chapter 2, the terms risk and uncertainty are used interchangeably.

²In this context, investment risk is equivalent to operating leverage or operating risk which is typically measured as the amount of fixed operating costs which result from the presence of intermediate and fixed assets on the balance sheet (128, pp. 771-779 and 130, p. 53).

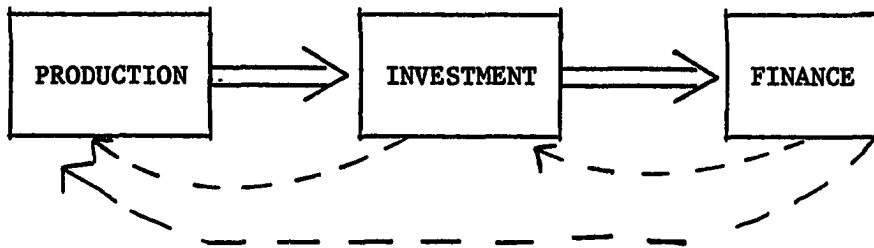


Figure 11. The theory of the firm and the interaction of the micro theories of production, investment and finance

Although the major sequence of decisions flows from production through investment to finance, in reality resource availability, resource prices and operating risks all influence the production decision. This feedback of the investment upon production is shown in Figure 11 as the broken line from the investment decision to the production decision. Similarly, the terms and conditions of financing influence which assets should be acquired.

Most economists recognize the interrelationship of production risk and investment risk. While most economists agree there is financial risk, some economists maintain that the financing decision (and its related risk) is independent of production and investment. An in-depth analysis of whether financing matters will be delayed until a later section. For the moment, assume financing does matter and, as portrayed in Figure 11, whole farm planning requires the joint attention to all three microtheories.

Flow of Funds Within the Firm

The owners of a family farm possess a certain amount of wealth in the form of durable and nondurable personal and business assets. In addition, the heirs may own or desire to own business and personal assets. Graphically, as in Figure 12, some proportion of the parents' wealth is held in productive business assets, and the balance is retained in the form of personal assets. The parents' investment in the farm is

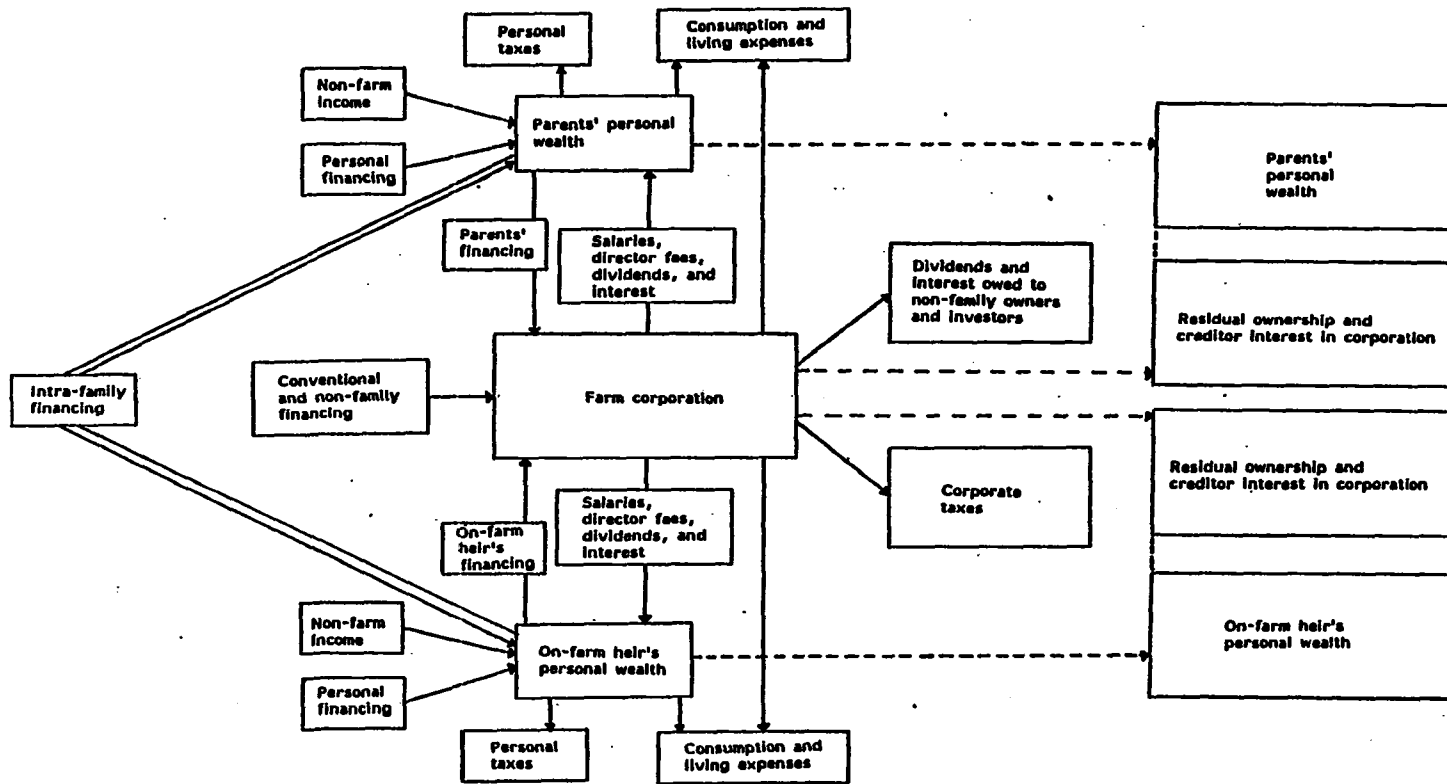


Figure 12. The flow of funds between family members and the farm corporation

increased by sources of nonfarm income and funds generated from personal financing, and these funds may be used to increase their contribution to the business. In return for these contributions, the parents receive either an equity interest in the form of common stock or contractual obligations on loans, rental agreements on leases or certificates on indebtedness of bonds.¹ The parents can receive salaries, directors' fees, dividends, interest and principal payments as compensation for their contributions to the firm. Leakages from the parents' wealth are expenditures for personal income taxes and personal consumption, some proportion of which may be paid by the firm.

The personal cash flows of the parents can be reinvested in new personal assets, used to payoff personal debts, or contributed to the family business. At the end of each accounting period, the parents' wealth is measured as the value of the personal assets owned, the present value of all financial obligations owed to the parents, and the value of their residual ownership in the family farm.

Similarly, the heirs to the farm may possess some initial endowment of wealth which is proportioned between contributions to the business and personal assets. The heirs can also contribute funds or physical assets to the firm in exchange for shares of common stock, bond and debenture certificates, or loan commitments.

¹For the reasons developed in Chapter 1, the legal form of organization is assumed to be the regularly taxed corporation.

Personal income taxes and personal consumption expenditures drain the heirs' personal wealth. Sources of nonfarm income and compensation for contributions in the forms of salaries and directors' fees, interest, dividends and principal payments increase the wealth of the heirs and can be funneled back to the firm as new contributions. In addition to intrafamily financing, the farm can also acquire funds from nonfamily sources such as banks, neighbors and friends or other outsiders.

The farm corporation represented in Figure 12 is shown in detail in Figure 13.

Initially, parental and heir financing is used to create a stock of assets for the firm. These assets can be augmented by conventional nonfamily financing and by nonbalance sheet methods, such as the acquisition of assets through operating leases and rental agreements. This creates a pool of farm production assets. The production process converts the farm production assets into commodities which may be sold for cash, accounts receivable, or stored in inventory for later sale. Accounts receivable will hopefully be converted into cash at some later date. Until that time receivables appear as an asset on the ending balance sheet and can be used as collateral for additional borrowing. Commodities held in inventory will appear as an asset on the ending balance sheet and can be used as collateral to support additional financing.

The cash generated from the sale of commodities, the liquidation of receivables, and the sale of inventory is used to pay operating expenses,

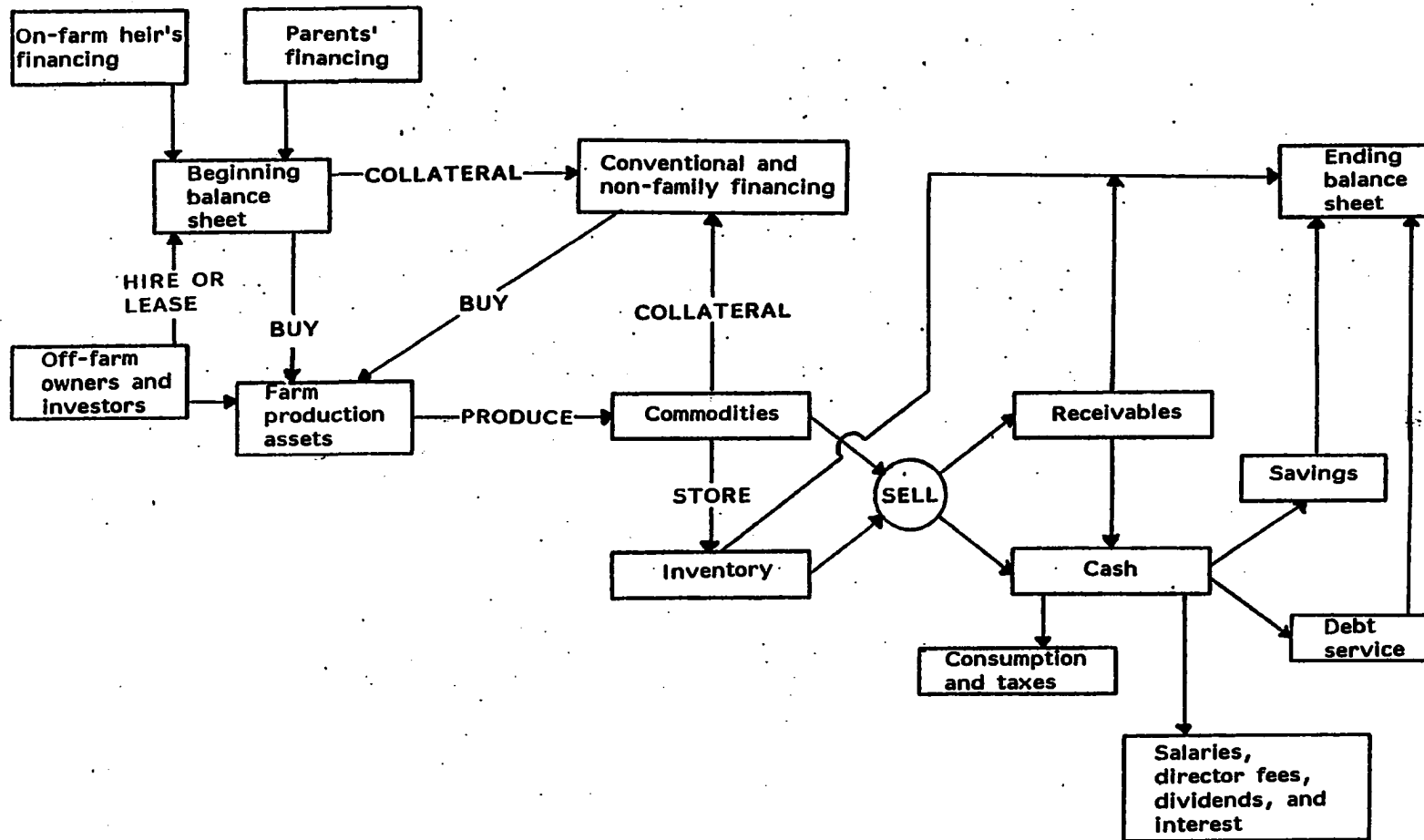


Figure 13. The flow of funds within the firm

salaries, directors' fees, dividends, interest and principal repayments on liabilities, corporate income taxes, consumption expenditures of the family members, and new assets. The balance sheet at the completion of the production cycle then becomes the beginning balance sheet of the subsequent period, and the cycle repeats itself. At several points, uncertainty influences the flow of funds within the firm.

The Components of Risk Which Comprise Total Enterprise Risk

The theory of the firm under uncertainty requires addressing risk in terms of total enterprise risk. Total enterprise risk is the multiplicative combination of the three risk components identified in Figure 11. The first component of risk is attributable to everything external to and uncontrollable by the firm. Uncertain events contributing to this category of risk include climate and weather variability, uncertain demand, inflation and uncertainty as to government intervention and policies.

The second component of risk is operating risk. Operating risk is attributable to the presence of fixed operating costs in the production process.¹ And the third component of risk is financing risk due to the presence of fixed financing costs.

¹The risk components of production and investment are often combined into a measure called business risk. Although the two approaches are equivalent, it facilitates the discussion to separate business risk into its components.

Production risk

Risk which is primarily uncontrollable and external to the farm is classified as production risk. The effect of this component on total enterprise risk is captured by stochastic prices of outputs and stochastic quantities of output. Mathematically total revenue is defined as

$$TR = \sum_{i=1}^n p_i q_i, \quad (4-1)$$

where

TR is total revenue,

p_i is the stochastic price of output i , and

q_i is the stochastic quantity of output i .

Total revenue is assumed to be normally distributed with a mean of $E(TR)$ and a variance $V(TR)$.¹ Production risk is measured as the variance of total revenue.

Variable operating costs are also assumed to be random because input prices and input quantities are also stochastic in nature. Variable operating costs are defined as

¹In reality, total revenue may or may not be distributed normally. In the presentation of this chapter, the question of normality need not be addressed and normality will be assumed to facilitate the discussion.

$$\text{VOC} = \sum_{j=1}^n r_j x_j, \quad (4-2)$$

where

VOC is variable operating costs,

r_j is the stochastic price of input j , and

x_j is the stochastic quantity of input j .

Earnings before interest and taxes (EBIT) can be defined as the difference between total revenue and variable operating costs and fixed operating costs (FOC), or

$$\text{EBIT} = \text{TR} - \text{VOC} - \text{FOC}. \quad (4-3)$$

As a simplifying assumption, assume that variable operating costs are a constant proportion of total revenue or

$$\text{VOC} = b \cdot \text{TR}. \quad (4-4)$$

Substituting (4-4) into (4-3) and rearranging terms, (4-3) can be rewritten as

$$\text{EBIT} = (1-b) \cdot \text{TR} - \text{FOC}. \quad (4-5)$$

The expected value of earnings before interest and taxes and its variance are calculated as

$$E(\text{EBIT}) = (1-b) E(\text{TR}) - \text{FOC}, \text{ and} \quad (4-6)$$

$$V(\text{EBIT}) = (1-b)^2 V(\text{TR}). \quad (4-7)$$

Since the constant b and fixed operating costs must be greater than zero to insure that a nonsensical solution does not result, the expected value of earnings before interest and taxes will be less than the expected value of total revenue. The variance of earnings before interest and taxes will be less than the variance of total revenue. But what is really important is the measure of relative variability and not absolute variability. Defining the coefficient of variability as

$$CV(\text{TR}) = \frac{\sqrt{V(\text{TR})}}{E(\text{TR})} \text{ and } CV(\text{EBIT}) = \frac{\sqrt{V(\text{EBIT})}}{E(\text{EBIT})}, \quad (4-8)$$

Then what is really of interest is the ratio of the coefficients of variation, or

$$\frac{CV(\text{EBIT})}{CV(\text{TR})} = \frac{\sqrt{V(\text{EBIT})/E(\text{EBIT})}}{\sqrt{V(\text{TR})/E(\text{TR})}} = \frac{\sqrt{V(\text{EBIT})}}{E(\text{EBIT})} \frac{E(\text{TR})}{\sqrt{V(\text{TR})}}. \quad (4-9)$$

Substituting (4-6) and the square root of (4-7) into (4-9) results

in

$$\begin{aligned} \frac{CV(\text{EBIT})}{CV(\text{TR})} &= \left[\frac{(1-b) V(\text{TR})}{(1-b) E(\text{TR}) - \text{FOC}} \right] \left[\frac{E(\text{TR})}{\sqrt{V(\text{TR})}} \right] \\ &= \frac{(1-b) E(\text{TR})}{(1-b) E(\text{TR}) - \text{FOC}} \end{aligned} \quad (4-10)$$

which must be greater than 1 as long as fixed operating costs are greater than zero. Furthermore, the larger are the fixed operating costs, the larger will be the relative variability of earnings before interest and taxes. This can be seen by taking the partial derivative of (4-10) with respect to fixed operating costs, or

$$\frac{\partial}{\partial \text{FOC}} = \frac{(1-b) E(\text{TR})}{[(1-b) E(\text{TR}) - \text{FOC}]^2} > 0 \quad (4-11)$$

which is positive as long as $b < 1$ and the firm is making a profit. An increase in fixed operating costs increases the ratio of the relative dispersions. Therefore, the presence of fixed operating costs increases the risk exposure of the firm already present from price and quantity variability. For any given degree of variability due to price and quantity uncertainty, an increase in fixed operating costs will increase multiplicatively the business risk and the total enterprise risk of the firm.

Fixed operating costs (and therefore operating risk) are primarily due to the presence of intermediate and long-term assets in the balance sheet. These are assets which are not consumed in one production cycle and generate depreciation and amortization charges against income. The composition of assets is primarily attributable to the production technologies employed by the firm and the expansion path the firm is following. Operating risk is a short run concept because in the long run (by definition) there can be no fixed costs.

The third component of risk to the firm is financial risk or financial leverage. Since total revenue is assumed to be normally distributed, earnings before interest and taxes from (4-6) and (4-7) must also be normally distributed. That is

$$EBIT \sim N(E(EBIT), V(EBIT)). \quad (4-12)$$

Net income before taxes is defined as

$$NIBT = EBIT - \sum_{k=1}^k r_k D_k, \quad (4-13)$$

where

NIBT is net income before taxes,

r_k is the interest rate on debt of type k, and

D_k is the amount of debt of type k employed.

The expected value and the variance of net income before taxes are defined as

$$E(NIBT) = E(EBIT) - \sum_{k=1}^k r_k D_k, \text{ and} \quad (4-14)$$

$$V(NIBT) = V(EBIT). \quad (4-15)$$

The coefficients of variation of net income before taxes (NIBT) and earnings before interest and taxes (EBIT) are

$$CV(EBIT) = \frac{\sqrt{V(EBIT)}}{E(EBIT)}, \text{ and} \quad (4-16)$$

$$CV(NIBT) = \frac{\sqrt{V(NIBT)}}{E(NIBT)} = \frac{\sqrt{V(NIBT)}}{E(EBIT) - \sum r_k D_k} \quad (4-17)$$

Dividing (4-16) by (4-17) and simplifying results in

$$\begin{aligned} \frac{CV(NIBT)}{CV(EBIT)} &= \left[\frac{\sqrt{V(EBIT)}}{E(EBIT) - \sum r_k D_k} \right] \left[\frac{E(EBIT)}{\sqrt{V(EBIT)}} \right] \\ &= \frac{E(EBIT)}{E(EBIT) - \sum r_k D_k} \quad (4-18) \end{aligned}$$

The expression in (4-18) must be greater than 1 with the presence of fixed interest charges so that the relative dispersion of earnings increases with fixed financing costs. The partial derivative of (4-18) with respect to fixed financing costs is

$$\frac{\partial}{\partial (r_k D_k)} = \frac{E(EBIT)}{(E(EBIT) - \sum r_k D_k)^2} > 0 \quad (4-19)$$

which says that as the amount of fixed financing costs increase, so does the ratio of the relative dispersions. Therefore, an increase in fixed financing costs increases financial risk and the total enterprise risk.

Financial risk is a function of the liability side of the balance sheet. If the liabilities are perpetual in nature (either perpetuities or liabilities of fixed term that are rolled over at maturity to generate new liabilities), then financial risk is both a short run and a long run concept.

The three sources of risk are multiplicative in nature and the farm should be concerned with the product of the three and not necessarily the particular level of any one. For a firm operating efficiently with a desired level of total enterprise risk, it is possible to substitute one source of risk for another and maintain the same level of total risk exposure. For example, the firm could employ such risk management strategies as diversification, hedging, spreading sales, forward price contracting, and insurance to decrease the variability in output prices and yields. Alternatively, the firm could reduce its operating leverage by substituting inputs which generate variable operating costs for inputs which lead to fixed operating costs. The substitution of labor inputs for capital and renting instead of buying are examples which decrease operating risk. Financial risk can be decreased with the substitution of equity capital for nonequity sources of funds. To be certain, all risk management strategies also influence the level of expected return such that a trade-off typically exists between return and risk.

The three components of enterprise risk suggest that the joint attention to production, investment and financing is necessary in whole farm planning. A great deal of literature has been written on the integration of investment and production decisions in agriculture (3, 32, 33, 53, 54, 57, 84, 87, 108). Considerably less has been written on the integration of financing with production and investment. This is partly

due to the unresolved question of whether financing matters. Therefore, it is now appropriate to discuss the theory of capital structure.

Theory of Capital Structure

The primary concern of the theory of capital structure is whether the way in which investments are financed influences the value of the firm (128, p. 261). For the sake of simplicity, assume there is only one type of debt available to the firm and only one type of equity. Furthermore, assume that the investment and production decisions have already been made and all that remains is to determine the most desirable combination of equity and debt.¹ If the ratio of debt-to-equity (leverage) matters, then the firm can affect its total valuation by changing its financing mix.

So that the analysis which follows can be more concisely presented, several facilitating assumptions will be made. Namely: 1) there are no income taxes (at least initially); 2) leverage is changed by issuing debt to repurchase stock or issuing stock to pay off debt; 3) changes in the capital structure are instantaneous with no transfer costs; 4) all

¹Following the finance school, "most desirable" means the maximization of share price. For a large corporation with an active secondary market for its stock, this is equivalent to saying all stock owners possess an increasing utility function (or $u' > 0$) which is the same as first degree stochastic dominance. Since the shares of stock of farms are typically not publicly traded because of their small size and private holdings, we will substitute the maximization of net total assets (total assets minus total liabilities) valued at current fair market values as a proxy for the maximization of market price. This is a realistic proxy as long as shares of common stock in the farm corporation accurately represent the economic value of the underlying farm assets.

earnings are paid in dividends; 5) the probability distribution of future earnings is the same for all companies and are known by all investors in the market; and 6) the operating earnings are not expected to grow--that is, the probability distribution of earnings is the same for every period.

Following the analysis developed by Solomon (115), the following three rates are defined

$$k_i = \frac{F}{B}, \quad (4-20)$$

where

k_i is the yield on the company's debt, assuming this debt is perpetual,

F is the annual interest charges on debt,

B is the market value of debt outstanding.

$$k_e = \frac{E}{S} \quad (4-21)$$

where

k_e is the required rate of return for investors,

E is the earnings available to common stockholders, and

S is the market value of stock outstanding.

With 100 percent dividend payout, no taxes and no growth, the value (k_e) represents the market rate of discount which equates the present

value of the stream of expected future dividends with the current market price of the stock (128, p. 263).

Finally, define

$$k_o = \frac{O}{V}, \quad (4-22)$$

where

k_o is the overall capitalization rate of the firm,

O is net operating earnings, and

V is total market value of the firm.

The overall capitalization rate can be equivalently defined as the weighted average cost of capital where $V = B + S$, or

$$k_o = k_i \left(\frac{B}{B+S} \right) + k_e \left(\frac{S}{B+S} \right).^1 \quad (4-23)$$

The concern with whether financing matters is then a question of what happens to k_i , k_e and k_o when the degree of leverage (as denoted by the ratio B/S) changes.

Durand (31, pp. 91-116) has proposed two approaches to the valuation of earnings--the net income approach and the net operating income approach.

¹For an analysis of the use of the three weights in (4-20), (4-21) and (4-22) in calculating a weighted average cost of capital, see Nantell and Carlson (91).

Net income approach to optimum capital structure

To illustrate the net income approach of Durand, an example from Van Horne (128, p. 264) is used where the hypothetical firm has \$3,000 of perpetual debt at 5 percent interest, the expected value of annual net operating earnings is \$1,000 and the equity capitalization rate, k_e , is 10 percent. The value of this firm to the stockholders is \$11,500--as is calculated in Table 1, column A.

The implied overall capitalization rate from (4-22) is

$$k_o = \frac{0}{V} = \frac{1,000}{11,500} = 8.7\%.$$

Now assume this hypothetical firm increases its debt from \$3,000 to \$6,000 and uses the cash proceeds to repurchase stock (the interest rate remains at 5 percent). The value of the firm increases to \$13,000--as shown in Table 1, column B. The implied overall capitalization rate is now

$$k_o = \frac{0}{V} = \frac{\$1,000}{\$13,000} = 7.7\%.$$

In this example, the hypothetical firm is able to increase the total value of its stock and decrease its overall capitalization rate by increasing its debt to equity ratio. The per share market price has now increased as a result. Because (initially assume there were 850 shares outstanding valued at \$10 per share) the \$3,000 in debt issued is used to

Table 1. Net income approach to capital structure^a

		-----Company-----	
		<u>A</u>	<u>B</u>
O	Net operating earnings	\$ 1,000	\$ 1,000
F	Interest (at 5%)	<u>150</u>	<u>300</u>
E	Earnings available to common stockholders	\$ 850	\$ 700
k_e	Equity capitalization rate	<u>.10</u>	<u>.10</u>
S	Market value of stock	\$ 8,500	\$ 7,000
B	Market value of debt	<u>3,000</u>	<u>6,000</u>
V	Total value of the firm	\$11,500	\$13,000

^aFrom Van Horne (128, p. 264).

purchase 300 shares of stock, leaving the firm with 550 shares. The value of the remaining common stock is \$7,000-- which, when divided by 550 shares outstanding, is equal to \$12.73 per share.

Graphically, this phenomenon is shown in Figure 14 where the degree of leverage employed is measured on the horizontal axis and percentages on the vertical axis. Since debt funds are cheaper than equity funds, the optimum capital structure is to employ as much debt as is institutionally possible (i.e., move as far to the right in Figure 14 as the firm can). The significance of the net income approach is that the firm can increase its value to its shareholders by increasing the use of debt funds.

The critical assumptions upon which the net income approach has been challenged are that k_f , and more importantly k_e , remain unchanged as the degree of leverage changes. These assumptions imply that the firm does not become more risky in the minds of investors and creditors as the degree of leverage is increased.

Net operating income approach to capital structure

At the opposite extreme of the net income approach is the net operating income approach. Under this theory, it is the overall capitalization rate, k_o , which remains constant regardless of the degree to which debt funds are employed. Using the same hypothetical firm as an example with $k_o = 10\%$, $O = \$1,000$ and $B = \$3,000$ at 5%, the implied equity capitalization rate is

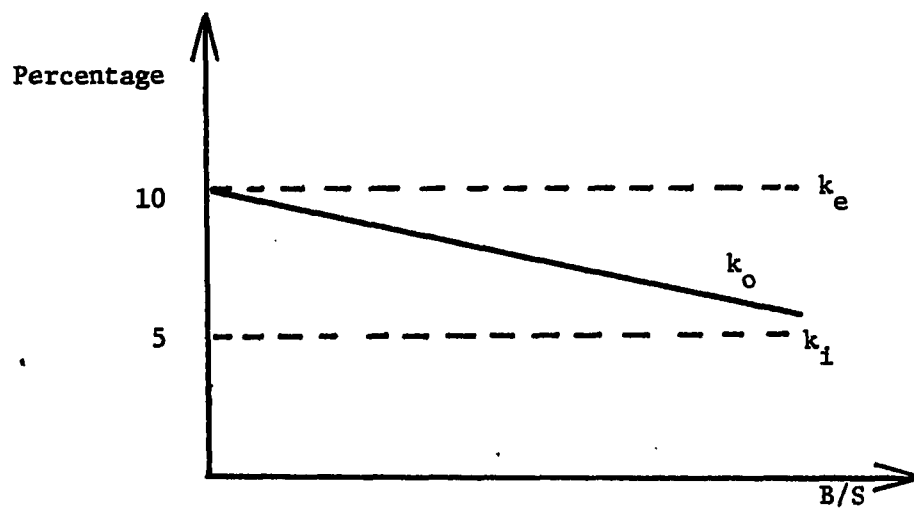


Figure 14. Net income approach to capital structure

$$k_e = \frac{E}{S} = \frac{\$850}{\$7,000} = 12.1 \text{ percent}$$

where S is calculated in Table 2, column A. Now suppose, as before, the hypothetical firm replaces \$3,000 of equity with \$3,000 of debt. The implied equity capitalization rate increases to

$$k_e = \frac{E}{S} = \frac{700}{4,000} = 17.5 \text{ percent}$$

where S is calculated in Table 2, column B. The equity-capitalization rate rises with increases in debt, but the total value of the firm remains unchanged. Graphically, the net operating income approach is shown in Figure 15. The graph reveals that the overall capitalization rate of the firm does not change with changes in leverage. What does change is the required rate of return on equity. Under this hypothesis, investors demand a higher return (lower price to earnings ratio) on their investment for an increase in leverage. Because the cost of capital is constant, any capital structure is as good as any other, and no unique optimum exists. To see this, assume again that there are initially 850 shares outstanding. The market value per share is then $7,000/850$ (from Table 2, column A) or \$8.23. The \$3,000 of debt is used to purchase 364 shares at \$8.23 each. Therefore, the market share price after the change (from Table 2, column B) is $\$4,000/(850-364) = \8.23 , the same as before. The investor is indifferent as to which capital structure is employed. The critical assumption in this approach is that the increase in the

Table 2. Net operating income approach to capital structure^a

		-----Company-----	
		<u>A</u>	<u>B</u>
O	Net operating income	\$ 1,000	\$ 1,000
k_o	Overall capitalization rate	<u>.10</u>	<u>.10</u>
V	Total value of firm	\$10,000	\$10,000
B	Market value of debt	<u>3,000</u>	<u>6,000</u>
S	Market value of stock	\$ 7,000	\$ 4,000
k_e	Equity capitalization rate (O-F)/S	12.1	17.5

^aFrom Van Horne (128, p. 264).

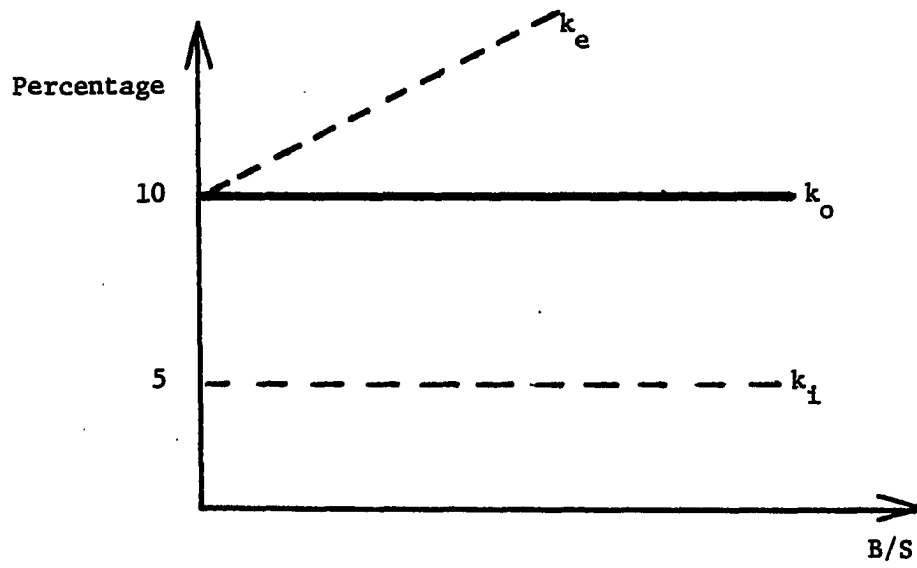


Figure 15. Net operating income approach to capital structure

required rate of return on equity is just exactly sufficient to offset the increased use of cheaper debt funds.

Major support was given to the net operating income approach by Franco Modigliani and Merton Miller in 1958 (95). They offered behavioral justification for a constant overall capitalization rate regardless of the degree of leverage employed. The assumptions upon which Modigliani and Miller based their argument are (128, pp. 270-271):

1. Perfect capital markets; perfect and free information; no transaction costs; perfectly divisible securities; and all investors behave rationally.
2. All investors view the expected probability distribution of operating earnings the same, and the distribution doesn't change over time.
3. Corporate income taxes and bankruptcy costs are absent.

Modigliani and Miller argue that arbitrage in the capital markets (and more importantly the arbitragers ability to substitute personal leverage for corporate leverage) will insure that two firms identical in every respect except their capital structure will be valued the same. But for arbitrage to be effective, the two firms must be viewed as identical substitutes by investors. In Modigliani and Miller's article, the two firms must be in the same risk class.¹

¹Modigliani and Miller propose to classify firms according to the degree of business risk to which they are exposed. Their arguments on arbitrage were then couched in terms of comparing two firms belonging to the same risk class. Subsequent authors have shown that this is an unnecessary restriction. For example, see Van Horne (128, pp. 292-294) and Becker (9, pp. 65-69).

Consider two firms belonging to the same risk class and identical in every respect except that A is not leveraged while B has \$3,000 of 5% bonds outstanding as in Table 3. The total value of Company B is greater than Company A. Modigliani and Miller maintain that this cannot happen because arbitragers will enter the market and sell shares of B to buy shares of A because they can obtain (with A's stock) the same dollar return with no increase in risk for a smaller investment outlay. This arbitrage would continue until the per share price of B's stock declined and the per share price of A's rose to the point where the total values of the two firms were identical.

Modigliani and Miller illustrate how this would occur (96). Suppose a rational investor owned 1 percent of Company B in Table 3. This ownership is worth \$77.27. The investor would practice arbitrage by: 1) selling the B stock for \$77.27; 2) borrowing \$30 (1% of B's corporate debt) at 5 percent; and 3) buying \$100 (1 percent) of A's stock. Prior to this series of transactions, the investor's expected return on B's stock was $\$77.27 \times .11 = \8.50 . After the transaction, his return on A's stock is $\$100 \times .10 = \10.00 from which he must subtract personal debt servicing charges of \$1.50 ($\$30 \times .05 = \1.50). His net dollar return on A's stock is therefore \$8.50, the same as it was on B's stock. However, his investment in B's stock was \$77.27 whereas his investment in A's stock is only \$70 (\$100 stock purchase minus \$30 of debt). Conversely, the investor was earning 11 percent on his investment in B's stock. After arbitrage the investor is able to earn 12.1 percent on A's stock

Table 3. Modigliani and Miller and the cost of capital^a

		-----Company-----	
		<u>A</u>	<u>B</u>
O	Net operating income	\$ 1,000	\$ 1,000
F	Interest on debt	<u>0</u>	<u>150</u>
E	Earnings available to stockholders	\$ 1,000	\$ 850
k_e	Equity capitalization rate	<u>.10</u>	<u>.11</u>
S	Market value of stock	\$10,000	\$ 7,727
B	Market value of debt	<u>0</u>	<u>3,000</u>
V	Total value of the firm	\$10,000	\$10,727
k_e	Implied overall capital rate	.10	.093
B/S	Debt-to-equity ratio	0	.388

^aFrom Van Horne (128, p. 272).

partially financed by personal debt. In essence, the investor is substituting personal leverage for corporate leverage by taking on personal debt. The concerted actions of all rational investors will drive up the price of Company A's stock and lower its k_e and drive down the price of B's stock and increase B's k_e . This will continue until the total values of the two firms is identical. As a result, the overall capitalization rates, k_o , must also be the same which is consistent with the net operating income approach. Conversely, if A's value exceeds B's value (A's equity capitalization rate is too low), arbitrage would occur in the opposite direction, raising A's k_e and lowering B's k_e until both firms are again valued the same.

Modigliani and Miller conclude that ¹

1. The total market value of the firm and its cost of capital are independent of its capital structure. The total market value of a firm is given by capitalizing the expected stream of operating earnings at a discount rate appropriate for its risk class.
2. The expected yield of a share of stock, k_e , is equal to the capitalization rate of a pure equity stream, plus a premium for financial risk equal to the difference between the pure equity capitalization rate and k_1 times the ratio B/S. In other words, k_e increases in a manner to exactly offset the use of cheaper debt funds.
3. The cutoff rate for investment purposes is completely independent of the way in which an investment is financed. This proposition, along with the first, implies a complete separation of the investment and financing decisions of the firm.

¹Restated from Van Horne (128, p. 271).

Obviously, pure adherents to the net operating income approach would find fault with the development of the theory of the firm at the introduction to this chapter. In particular, they would argue there are no decision making linkages between investment and finance nor between production and finance as pictured in Figure 11. However, the Modigliani-Miller theory of capital structure, in its simplest form, abstracts from the effects of market imperfections such as taxes, bankruptcy, and credit rationing. As will be seen in the next section, the inclusion of market imperfections makes the Modigliani-Miller theory consistent with the earlier presentation of the role of financing.

Traditional theory of capital structure

The traditional theory of capital structure includes all the ground between the net income and the net operating income approaches. But the traditional approach assumes that there is an optimum capital structure and that the firm can increase its total value through the judicious use of leverage (128, p. 268).

As an example of one variation of the traditional theory, consider the following. The hypothetical firm of the previous examples has \$1,000 in net operating income with no debt and an equity capitalization rate of 10 percent. The total value of this firm is presented in column A of Table 4. In the absence of leverage, the total value of the firm is \$10,000 with a per share price of \$8.23 and an overall capitalization rate of 10 percent. Now assume that the firm issues \$3,000 of debt at 5

Table 4. Traditional approach to capital structure

		-----Company-----		
		<u>A</u>	<u>B</u>	<u>C</u>
O	Net operating income	\$ 1,000	\$ 1,000	\$ 1,000
F	Interest on debt	<u>0</u>	<u>150</u>	<u>360</u>
E	Earnings available to common stockholders	\$ 1,000	\$ 850	\$ 640
k_e	Equity capitalization rate	<u>.10</u>	<u>.11</u>	<u>.14</u>
S	Market value of stock	\$10,000	\$ 7,727	\$ 4,571
B	Market value of debt	<u>0</u>	<u>3,000</u>	<u>6,000</u>
V	Total value of firm	\$10,000	\$10,727	\$10,571
C	Shares outstanding	1,214.3	850	520
S/C	Market value per share	\$ 8.23	\$ 9.09	\$ 8.79
k_o	Overall capitalization rate	.10	.093	.095

percent and uses the proceeds to purchase 364.3 shares of stock ($\frac{3,000}{8.23} = 364.3$). The investors now view the firm as more risky with the presence of debt in the capital structure and raise their equity capitalization rate from 10 percent to 11 percent. The total value of the hypothetical firm is calculated in column B of Table 4. The total market value has increased to \$10,727, the market price per share has risen to \$9.09, and the overall capitalization rate has dropped to 9.3 percent. Although k_e , the required rate of return on equity, increased with the increase in leverage, the increase does not entirely offset the benefit of using cheaper debt funds. As a result the overall capitalization rate has dropped.

Now suppose the firm increases its debt from \$3,000 to \$6,000, but the interest on debt also rises from 5 percent to 6 percent. Additionally, investors view the firm as more risky than before and increase their required rate of return on equity from 11 percent to 14 percent. The valuation of the firm is calculated in column C of Table 4. The additional \$3,000 of debt is used to purchase 330 shares at \$9.09 per share, which leaves 520 shares outstanding. The market value per share is now \$8.79 (lower than company B), and the overall capitalization rate is 9.5 percent (higher than company B). Thus, the total valuation of the firm is lower and the overall capitalization rate is higher when debt is increased from \$3,000 to \$6,000.

Graphically, this version of the traditional approach is portrayed in Figure 16. The cost of debt, k_d , remains constant to some point of

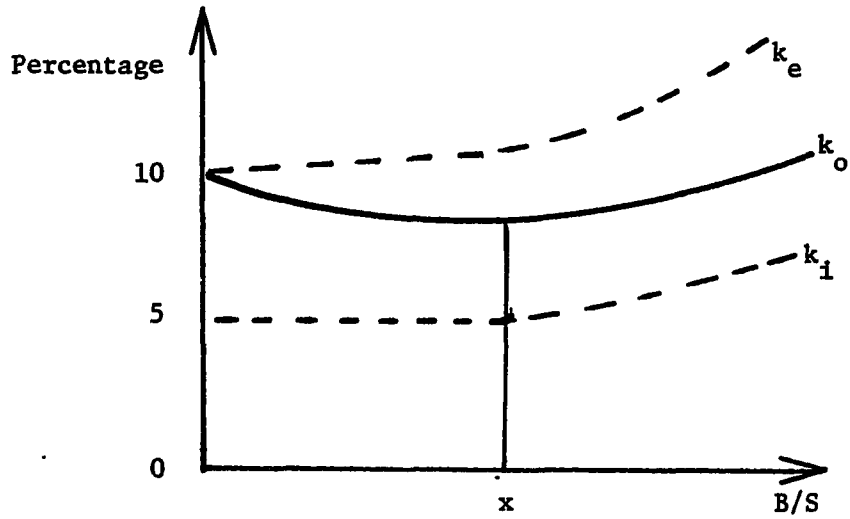


Figure 16. Traditional approach to capital structure

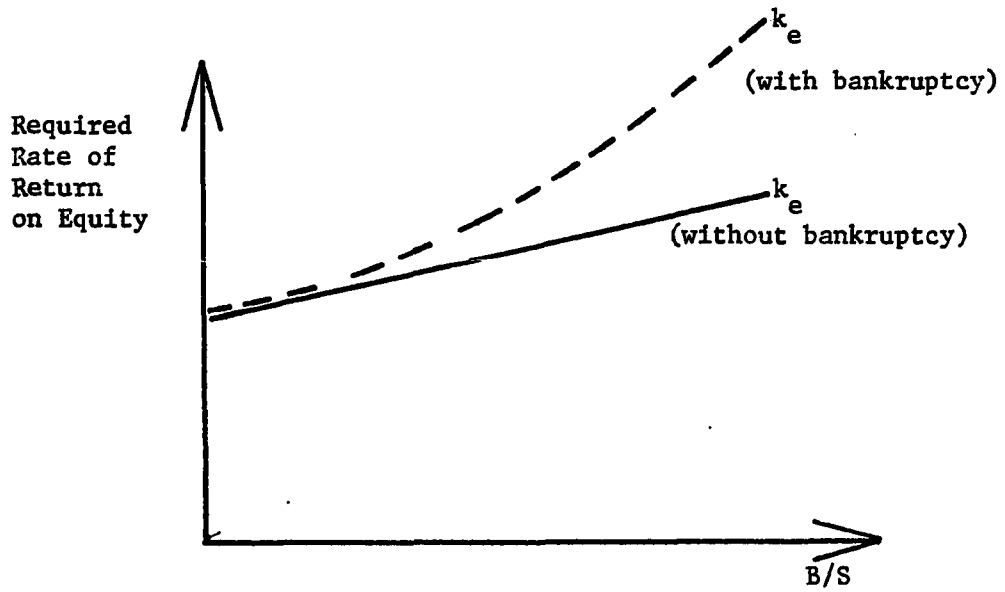


Figure 17. Bankruptcy costs and the cost of capital

leverage and begins to increase beyond that point because creditors view the firm as more risky and require a higher return on their loan funds. Investors also require compensation for additional risk taken on by the firm; therefore, k_e rises at an increasing rate with leverage. With a small use of leverage, the increase in k_e does not fully offset the use of cheaper debt funds, so initially the overall capitalization rate declines. But after some point (such as at X in Figure 16), the increase in k_e more than offsets the cheaper cost of debt, and the overall capitalization rate begins to increase. The increase in k_0 is supported further once the cost of debt k_1 also begins to rise. The optimal capital structure of the firm is the minimum point of k_0 which occurs at point X. At point X the marginal real cost of debt is equal to the marginal real cost of equity. The traditional approach (either this variant or some other) implies that the cost of capital is not independent of the capital structure; therefore, financing does influence investment and production.

Market imperfections and capital structure

The three theories of capital structure are extremely divergent in their conclusions. From a theoretical standpoint, it is difficult to choose among them. The choice must be made upon one's belief of the effect increasing leverage has on the cost of debt and the required return demanded by investors. In order to choose, the competing theories must be evaluated in light of economic behavior--that is, the theory which is most consistent with reality must be found. Theory is by

definition an abstraction of reality where the abstract nature is achieved through specification of simplifying restrictions. Modigliani and Miller assume that perfect capital markets exist. Consequently, to prove that the net operating income approach is not an appropriate application of theory, one must show that perfect markets are not a reasonable assumption. Furthermore, even if market imperfections exist, they must be shown to be significant (128, p. 274).

Transaction costs

One imperfection in the market place is the cost of transactions. Transaction costs do restrict the arbitrage process for equilibrating the market, but as Hirschleifer (58, pp. 264-275) has shown, arbitrage will still take place up to the limits of the transaction costs. Transaction costs result in imperfect markets, but the effect is not systematic as to direction so that the net effect (if it exists at all) is not predictable. Therefore, although transactions costs are an imperfection, they are not sufficient grounds for refuting the net operating income approach.

Bankruptcy costs

Under the assumptions of perfect capital markets, bankruptcy proceedings result in zero costs. Presumably, assets are liquidated at their economic values and the proceeds distributed at no cost to creditors and investors according to their claims. The creditors and investors are then able to reinvest the proceeds in equivalent endeavors.

Under perfect market restrictions, investors and creditors are indifferent to the event of bankruptcy. Haugen and Senbet (52, pp. 383-394) argue that even if a market imperfection due to bankruptcy exist, it is still not an important factor because when bankruptcy approaches, the firm simply reorganizes by selling stock and repurchasing debt. The firm can do this at minimum cost because all parties involved act rationally and share an incentive to avoid formal bankruptcy.

Van Horne states that although the evidence is fragmentary, the administrative costs of liquidation due to bankruptcy may approach twenty percent of the value of the assets (128, p. 226). This is in addition to any liquidation loss due to the sale of assets at less than their economic value. Security holders as a whole would receive less in the event of positive bankruptcy costs than in their absence. To the extent that a levered firm has a greater possibility of bankruptcy, investors would prefer an unlevered firm, all other things the same. Kraus and Litzenberger (71, pp. 911-922) show that the possibility of bankruptcy is a nonlinear function of leverage (i.e., beyond some threshold it increases at an increasing rate). As a result, expanding leverage would be expected to have a negative effect on the value of the firm and its cost of capital (129). Since bankruptcy costs are a dead-weight loss, investors are unable to diversify away this risk. Therefore, investors penalize the price of a stock (by requiring a higher k_e) as leverage increases. The effect of bankruptcy costs is shown in Figure 17, page 90. If the required return on equity is linear in the absence of

bankruptcy costs, as shown by the solid line in Figure 17, the arguments of Van Horne, Krause and Litzenberger would make k_e bend upward at an increasing rate with the addition of bankruptcy costs, as shown by the broken line in Figure 17. The additional return required by investors at low levels of leverage is negligible but increases at an increasing rate. At high leverage, the effect is substantial. Therefore, the cost of capital and leverage are not independent, and financing is not independent of investment.¹

Homemade leverage and the cost of borrowing

Another assumption upon which Modigliani and Miller's argument is based is that personal and corporate leverage are perfect substitutes. A number of market imperfections, however, indicate this is not the case. An individual is unlikely to negotiate the same borrowing terms as a corporation. In addition, individuals face unlimited liability on personal loans whereas corporate leverage results in liability only to the value of the stock. Margin calls and the personal time involved in facilitating personal loans may also make personal and corporate leverage less than perfect substitutes.

¹Joseph Stiglitz (119, pp. 851-866) suggests that bankruptcy affects the cost of capital even in the absence of bankruptcy costs. He makes this contention because of a divergence in expectations as to the chance of bankruptcy between lender and borrower. This results in an increasing interest rate (due to increasing leverage) and therefore scale becomes an important factor and the firm's investment and financing decisions are no longer independent.

However, arbitrage need not be done by individuals. Other corporations and financial intermediaries entering the market may ensure efficient functioning of the arbitrage process to equilibrate the market even if individuals can not (118, 119). Furthermore, individuals can accomplish arbitrage without actually borrowing funds. They can accomplish the same thing by adjusting their portfolios of bond holdings (128, p. 277). Therefore, these market imperfections appear to be of negligible importance.

Income taxes

Since interest payments on debt are tax deductible, leverage results in a tax shield benefit accruing to the residual owners of the firm, i.e., the stockholders. If the tax shield remains unchanged in all future years, the present value of the tax shield is

$$PV = \frac{t_c r B}{r} = t_c B, \quad (4-24)$$

where

t_c is the corporate tax rate,

r is the interest rate on debt, and

B is the market value of debt outstanding.

Therefore, the value of the firm with taxes is now

$$V = \frac{O(1-t_c)}{\rho_k} + t_c B, \quad (4-25)$$

where

ρ_k is the overall capitalization rate (after tax) of a firm with no debt in a given risk class, and

O is the expected net operating earnings.

The first expression on the right hand side of (4-25) is the value of an unlevered firm under taxes. The second term on the right is the additional value to the stockholders due to the deductibility of interest payments. In essence, the government is paying the levered firm a subsidy for using debt (128, p. 280). Additionally, the more debt employed, the greater the subsidy and the lower will be the cost of capital. Reworking the Modigliani and Miller approach to incorporate taxes suggests that the optimum capital structure is to use as much debt as possible.

However, the tax shield is only available if there is sufficient taxable income to offset it. Similarly, the future benefits of the tax shield will not be realized if bankruptcy should occur. Personal income taxes may also work to mitigate the tax advantage of debt to the corporation. Capital gains are subject to a lower personal income tax rate than is ordinary income. With a 100 percent dividend payout and no growth in earnings, no appreciation (and therefore no capital gains) would accrue to the owners of stock. However, as the corporation increases its leverage to take advantage of the tax shield at the corporate level, the overall capitalization rate declines and the per share price of stock increases, producing a capital gain. However, the

creditors of the corporation must report the debt income as ordinary income and pay a higher rate than on capital gains. Therefore, the overall affect of personal taxes is to reduce the tax benefit at the corporate level (128, pp. 281-282 and 79, pp. 737-749).

Restructuring the Modigliani and Miller argument to include taxes and bankruptcy costs is shown in Figure 18. With recognition given to the effects of corporate income taxes only, the cost of capital to the firm is linear and decreasing. Therefore, the firm would employ as much debt as it conceivably could and the optimum capital structure would lie as far to the right as possible. Incorporating the mitigating effect of personal income taxes, the cost of capital would still be linear and downward sloping, but with a larger slope (i.e., smaller negative slope). The optimum capital structure would still be at the very far right. With the addition of bankruptcy costs, the cost of capital curve in Figure 18 would at first coincide with the other curves. As leverage becomes more pronounced, the effect of bankruptcy would partially offset the tax effect and at extreme leverage, bankruptcy considerations would more than offset the tax effect so that, even under the Modigliani-Miller approach, an optimal capital structure would exist.

If one's beliefs dictate that other market imperfections, such as imperfect information and differential borrowing rates, are significant factors, then the overall capitalization rate would turn up sooner, resulting in a smaller degree of leverage in the optimum capital structure. For additional discussion of determining the optimal capital

structure of a firm in the face of positive taxes and bankruptcy costs, see Kim (69), Scott (112), Lee and Baker (76), Baron (7) and Chen (25).

If the importance of market imperfections in determining the optimal capital structure is accepted, then the net income approach, the net operating income approach and the traditional approach all produce an optimum combination of debt and equity. Fortunately, for the purpose here, it is not necessary to choose among them, for the only concern is to establish the linkage between financing and investment. With market imperfections, the linkage does exist and financing does indeed matter.

To this point, debt has been treated as a homogeneous commodity. In fact, it has implicitly been assumed to be perpetual in nature. Since this is an unrealistic abstraction, it is now necessary to turn our attention to the types of financing available to the family farm.

Conceptual Considerations for the Type of Financing

In the previous section, the importance of financing was addressed. With the presence of market imperfections (most notably income taxes and bankruptcy costs), an optimum capital structure does exist. However, the discussion was limited to two types of financing - debt and equity. But debt is not a homogeneous commodity because of differing maturities and financial characteristics. In terms of maturity, liabilities can be classified as current, intermediate, and long term. In terms of characteristics, liabilities can be loans, bonds, contracts, and leases,

just to name a few. Heterogeneity does not, in itself, lead to the conclusion that an optimum composition of financing exists. If capital markets were indeed perfect, equity owners would be indifferent to the type and maturity of the debt the firm employs because the firm could not affect its valuation by altering the composition of debt (128, p. 481). However, capital markets are not perfect and therefore, the composition and type of financing are important. In this section, the effects of market imperfections will briefly be reviewed and the salient characteristics of selected types of financing will be identified.

The most important imperfections affecting debt financing are flotation costs, bankruptcy costs, costs of information, restrictions on lenders and absence of market determined transfers. Flotation costs which are fixed, in whole or in part, tend to bias the financing process towards less frequent financing, larger offerings at each issue and longer maturities. This is because a fixed financing charge leads to economies of scale with respect to debt offerings (128, p. 482).

Bankruptcy cost considerations also bias structuring the repayment schedule toward lower levels of debt obligations coming due in the near future; that is, using debt with longer maturities. Costs of information limit the number of financing arrangements available to the firm. Most family farms are not of sufficient size to justify a debt offering on publicly traded markets. This is because the cost of information to the ultimate investors is sufficiently high to make the public offering

unfeasible. Instead, the family farm will negotiate directly with a single party such as a commercial bank, Farm Credit Bank, insurance company or family member.

Restrictions on lenders may also limit the number of financing options available. These restrictions may be of a legal nature, a result of tax considerations, or self-imposed. For example, commercial banks face loan limits on the maximum amount they can loan to any one individual or entity. Commercial banks also favor short and intermediate term financing because of restrictions on their investment behavior. Federal Land Banks, on the other hand, specialize in making long term real estate loans. Dealings with family members are limited by the financial resources and consumption needs of each individual. As a result, the family farm may face limited sources of funds of a given maturity and type. This may force the farm to employ a maturity and type composition that is suboptimal.

Due to the small size and limited number of participants in a family farm's financing market (that is, small in relation to publicly traded corporations), the terms of a debt obligation result from the direct negotiation of two parties. There is no efficient secondary market which determines effective interest rates or maturities. Rather, the conditions of the debt obligation are negotiable and often occur at less than arm's length. As a result of these imperfections, the farm can alter its value to its stockholders by the way it packages its financial instruments.

Maturity

One approach to the question of optimum maturity composition is the hedging approach. Under this approach, each asset is offset by a liability of the same approximate maturity. Short term variations in the level of current assets (possibly due to cyclical or seasonal fluctuations) would be financed with short term debt. Fixed assets and permanent current assets needed in the production process would be financed with long-term or permanent sources of funds. This relationship is shown in Figure 19. Over time, as the firm grows, so does the firm's use of long-term and permanent sources of funds. As a firm moves into a season of extra funds need, short-term financing would be used to acquire the additional inputs. As the firm progresses into a period when funds needs decline, short term borrowing would be paid off with surplus cash. In the short term borrowing troughs, the firm would have no short-term borrowing apart from current installments on long term debt obligations. In this manner, the firm would employ financing only when it is needed. Under conditions of certainty, an exact synchronization of needs with borrowings such as the hedging approach would be possible. Under uncertainty, this is no longer possible because seasonal funds needs, interest rates, and net cash flows can all deviate from their expectations. Typically, a firm will not structure its repayment schedule in such a fashion as to require payment before the cash flow is generated. In short, the firm will lag the repayment schedule to the generation of funds. The longer the lag, the larger the margin of safety the firm has in meeting its obligations.

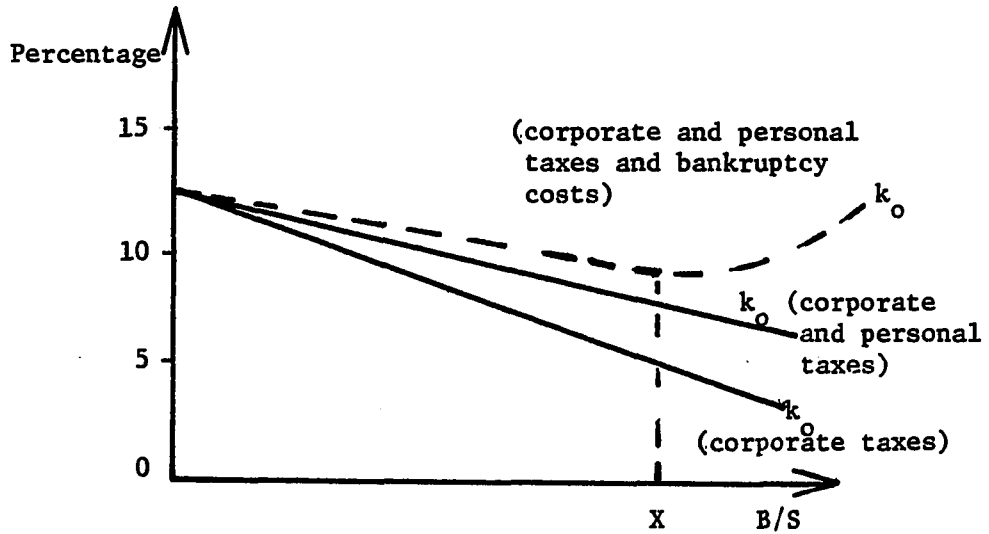


Figure 18. Net operating income approach with taxes

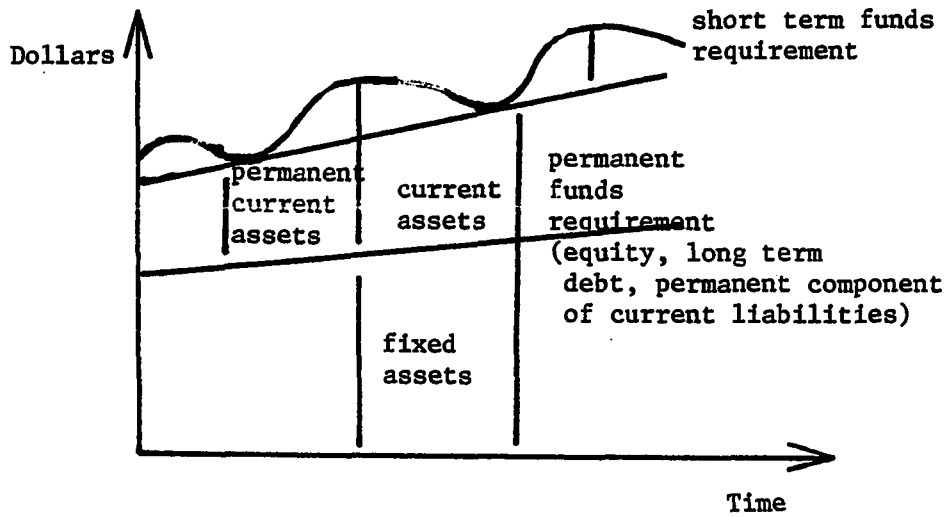


Figure 19. Hedging approach to financing

The degree of safety the firm desires is determined by a trade-off of profitability for reduced risk.

Long-term, fixed-rate financing provides the firm with a certain repayment schedule at a certain interest rate that is known at the time the loan is entered into. This provides the firm with more certainty (and therefore less risk) than does financing with an equivalent amount of short term debt. But this decrease in risk typically comes at a higher cost to the firm. Typically, the explicit interest cost of long-term debt is higher than short-term.¹ In addition, the firm will pay interest on funds when they are not needed.

Alternatively, with short term financing, the firm faces more uncertainty as to the ability to refinance and the cost at which refinancing will occur. This is of particular concern when long term assets are acquired with short-term credit. The cash flows from the income generating asset are not sufficient to pay off the short term loan. If for some reason the firm is unable to "roll over" the short term credit, the firm may be unable to meet the debt obligations and suffer short-run cash insolvency.

Even if refinancing is possible, there still remains the uncertainty as to interest costs. The obvious question that arises is --does a positive correlation between short term interest costs and net operating

¹It is possible at any point in time to have a downward sloping or humped yield to maturity curve, however, over a long enough period of time the firm typically pays more for long term than short term financing, particularly if the borrowings are privately negotiated (2, pp. 1249-1254).

income exist? From the end of World War II until 1970 there did exist a positive correlation between short term interest rates and corporate net income (128, p. 485). Over this period, both interest rates and profits tended to follow the business cycle. When corporate profits were depressed, so were interest rates; when profits were high, the rates were high also. Short-term interest rates acted to level out corporate income and reduce overall variability in corporate profits. But in the 1970s, inflation seems to have broken the correlation, for in 1970-71, 1975-76, and 1979-80, short-term interest rates remained high while profits were depressed. With the presence of high rates of inflation, it is unclear if any correlation exists between profits and interest rates.

The firm is then faced with the trade-off of higher interest costs at more certainty for possibly lower interest costs with more uncertainty. The decision on how much short-term and how much long-term debt to employ will be decided by the decision makers' preferences in terms of his risk-return tradeoff.

Current liabilities Current liabilities are defined as short term liabilities whose liquidation is reasonably expected to require the use of existing resources properly classified as current assets or the creation of other current liabilities (1, p. 21). The period "short term" is defined as within the next year or operating cycle, consistent with the definition of current assets (131, p. 140).

Current liabilities can be subdivided into two primary categories: those which arise due to temporary and seasonal financing needs, and

those which represent the current installments on intermediate and long term debt.

Examples of current liabilities which are due to temporary and seasonal borrowing needs are trade credit; accounts payable; accrual accounts such as wages, salaries, rentals, and expenses payable; revenues which have been collected in advance; and short term notes payable. The demand for these items closely follows the production process. When production is occurring at a high rate and generates a high demand for inputs, the demand for short-term funds rises. When production is at a low level, the demand for funds also slackens and the need for current liabilities diminishes. Because these changes are spontaneous, current liabilities are determined more by the level at which production is occurring and the desired level of working capital (current assets minus current liabilities) or short-term liquidity, than by discretionary management decisions. Typically, if payment is made within the credit terms, no interest cost is involved and these items represent a source of float to the firm.

Short-term notes payable do carry an explicit (or implicit, if sold at a discount) interest charge. In addition to short-term bank loans, these items may also include commercial paper, banker's acceptances, and other money market instruments.

The third component of current liabilities is the current installments due on intermediate- and long-term liabilities. These installments result from the payment structure of the intermediate- and long-term

liabilities. As a result, the level of current liabilities needed in any period can be described as the sum of the nondiscretionary funds requirements (that is, a fixed percentage, ϕ , of current assets) plus short term notes plus the current installments due on noncurrent liabilities or

$$CL_t = \phi \cdot CA_t + ST \text{ Notes}_t + \Sigma \text{ INSTALLMENTS}_t. \quad (4-26)$$

Intermediate and long term liabilities In accordance with generally accepted accounting principles, an intermediate- or long-term liability is an obligation that will not require the use of current assets for payment during the upcoming operating cycle or during the next year, whichever is longer (131, p. 143). It is not necessary to distinguish between intermediate and long term. In practice the two are often presented under the heading of long term. The two classes will be separated here merely for convenience and defined consistent with intermediate and long term assets where the time of demarcation is set at 5 to 6 years. What is important is the distinction between current and noncurrent liabilities and the maturities of the noncurrent liabilities.

Examples of financing instruments which fall within these two headings are conventional bank term loans where the bank may be a commercial bank, a credit union, a savings and loan, a Production Credit Association, a Federal Land Bank Association, or an insurance company.

Revolving bank credit, equipment-financing loans, and chattel mortgages are all intermediate- or long-term liabilities.

The common characteristic of intermediate- and long-term liabilities is that they are self-liquidating over a period in excess of one year. For repayment, the lender looks to the cash flow generating ability of the firm and the asset's collateral value (if the loan is secured). In addition to the monetary terms stipulated in the agreement, other conditions (protective covenants) may also be imposed by the lender. Some common protective covenants are requirements to maintain a stipulated level of working capital, restrictions on the payment of cash dividends, stock repurchase limitations, capital expenditure limitations, required insurance and restrictions on acquiring other indebtedness. The number and severity of covenants result from direct negotiation between borrower and lender.

Conventional term loans

All of the financing instruments described below will be classified as conventional term loans or term loans where four parameters (amount, repayment schedule, interest rate and maturity) are points of negotiation.

Upon agreement, the lender transfers cash proceeds to the borrower in return for a contractual obligation, as shown in Figure 20. The borrower uses the cash proceeds to acquire productive assets. The assets purchased, in conjunction with the existing stock of assets, are employed

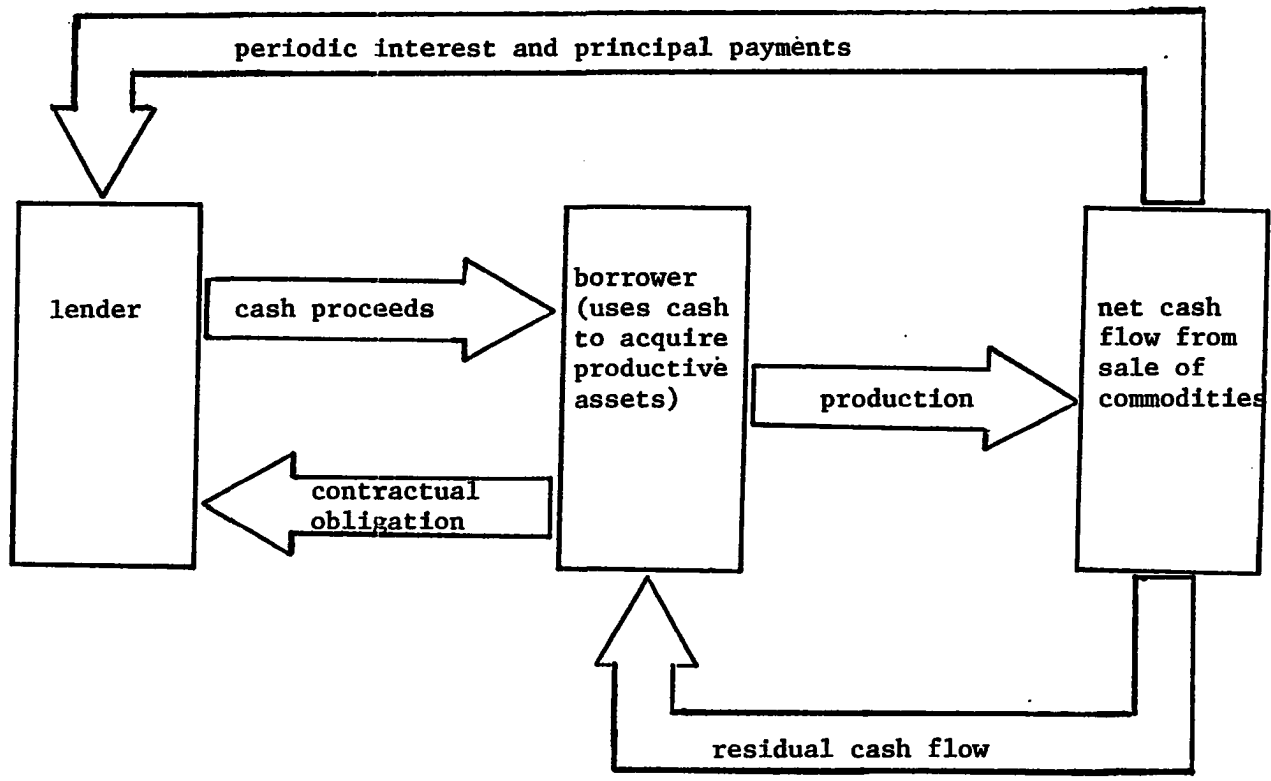


Figure 20. Conventional term loans

in the production process to produce commodities. The cash generated from the sale of commodities is used to pay the periodic interest and principal installments required by the obligation. Any residual cash flow is available to the borrower for consumption, dividends, or reinvestment in additional assets. If the net cash flow generated is insufficient to meet the debt-servicing needs of the obligation, then other sources of liquidity must be employed to meet the fixed financing costs.

The effect on the borrower's balance sheet at the time the loan is entered into is shown in Figure 21. Initially, current assets are increased by the cash proceeds. At a later time (or possibly instantaneously), the cash proceeds will be used to purchase assets. If the assets purchased are not current assets, then current assets will be decreased and the corresponding intermediate- or long-term asset accounts will be increased by the price of the purchase.

The present value of all future principal and interest payments discounted at the market rate of interest (assumed to be the same interest rate stipulated in the loan) is included as a long term liability. As each principal payment is made, the remaining obligation is revalued using the market rate of interest at the date of inception. Mathematically, the present value of the remaining obligation is computed as

$$PV_t = \sum_{i=1}^{n-t} \frac{(PR_{i+t} + INT_{i+t})}{(1+r)^i} \quad (4-27)$$

Assets	Liabilities
+ cash proceeds	+ present value of future interest and principal payments discounted at the the market rate of interest

Figure 21. Effect of taking out a loan on borrower's balance sheet

where

PV_t is the present value at the end of year t ,

n is the original life of the loan in years,

PR_{i+t} is the principal payment due in year $i+t$,

INT_{i+t} is the interest payment due in year $i+t$, and

r is the market rate of interest at inception.

Leases A lease is a means by which a firm acquires the economic use of an asset. The lessor transfers the property or the right to use the property, but not the title, to the lessee in return for a rental agreement as shown in Figure 22. The lessee employs the asset in his productive process to produce commodities which are then converted into cash. The net cash flow is used to make the periodic lease payments under the agreement. Any residual cash flow is retained by the lessee or any shortage of cash flow is made up from other sources.

For accounting purposes, a lease is classified as either an operating lease or capital lease.¹ A capital lease is broadly defined as a lease which transfers most of the risks and rewards of ownership from lessor to lessee or includes a "bargain purchase" option. An operating lease is any lease which does not qualify as a capital lease (131, p. 749). From the standpoint of the lessee, a lease must be

¹As defined in paragraph 6 of Financial Accounting Standards Board Statement Number 13, "Accounting for Leases" (38).

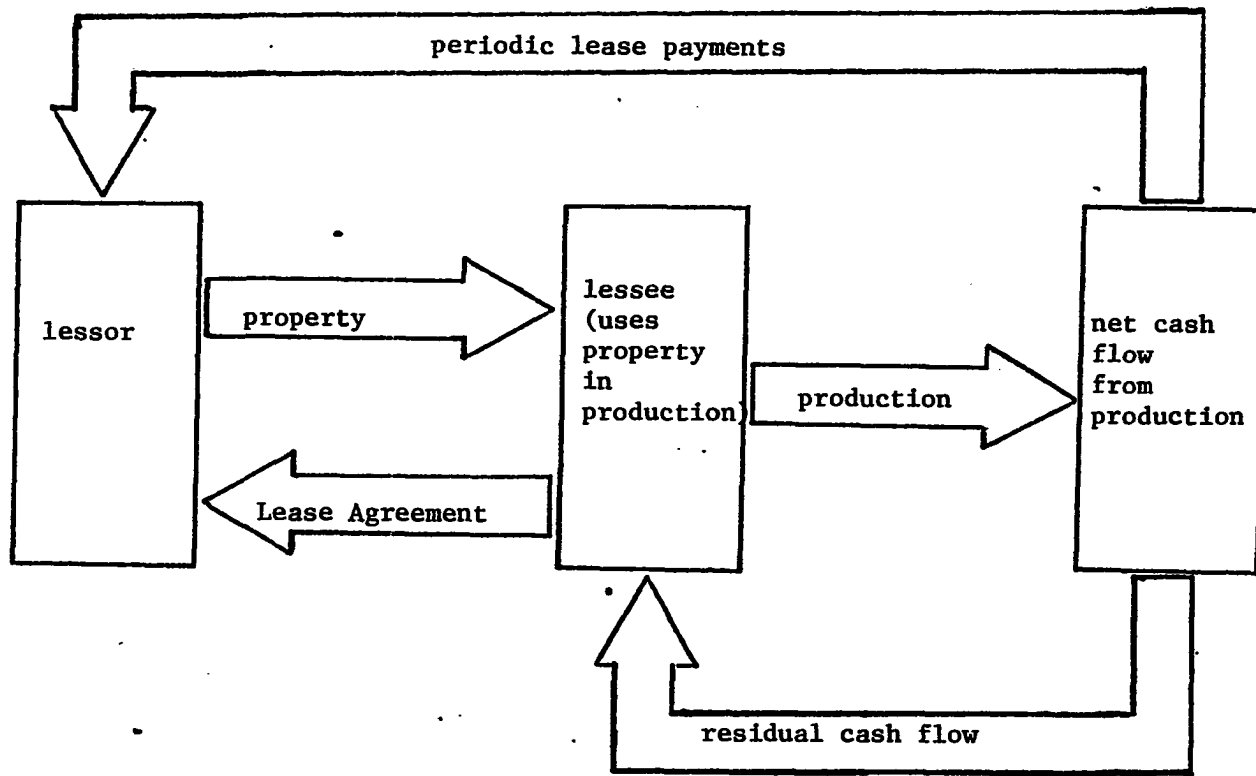


Figure 22. Capital and operating leases

classified as a capital lease if it meets one or more of the following conditions:

- 1) The lease transfers title to the lessee at or before the end of the contract.
- 2) The lease contains an option to purchase the asset at a bargain price.
- 3) The lease period equals or exceeds 75% of the asset's economic life.
- 4) The present value of the minimum lease payment stream equals or exceeds 90% of the net value realized by the lessor. The rate of discount is the lesser of the lessee's incremental borrowing rate or the lessor's internal rate of return (provided the latter can be calculated). The net value realized by the lessor is the asset's fair market value less any investment tax credit claimed by the lessor.

If the lease fails all four of the above tests, then by default, it is considered an operating lease.

The effect of a capital lease on the balance sheet of the lessee is shown in Figure 23. Both the asset and liability sides are increased by an amount equal to the present value of the lease payments but not to exceed an amount in excess of the asset's fair market value. At inception, the present value of a capital lease is calculated as

$$PV = \sum_{i=1}^n \frac{\text{Rent}_i}{(1+r)^i}, \quad (4-28)$$

CAPITAL:

Assets	Liabilities
+ present value of minimum lease payments including any bargain purchase option discounted at lessee's incremental rate of borrowing but not to exceed the property's fair market value.	+ same as asset side

OPERATING:

Assets	Liabilities
+ prepayment (if any) (terms of lease exceeding one year must be disclosed in a footnote or supporting schedule)	+ prepayment (if any)

Figure 23. Effect of a lease on lessee's balance sheet

where

$Rent_1$ is the lease rental due in year 1 of an n year lease,
and

r is the lessee's incremental rate of borrowing at the date of inception or, if practicable to calculate, the lessor's internal rate of return on the asset if this rate is less (38, paragraphs 5, 7).

The lease is amortized (both asset and liability) in a manner consistent with the firm's depreciation policy on similar assets, but the period of amortization must be the lease term rather than the life of the leased property (38, paragraph 11).¹ For a firm using straight line depreciation on equivalent assets and a lease which requires equal periodic rentals over the term of the lease, the annual reduction to the asset and liability entries will be the per period lease payment minus interest on the outstanding lease obligation. Interest, in this case, is calculated as the discount rate used in the present value computations times the account balance at the beginning of the period.

An operating lease does not affect either the lessee's or lessor's balance sheet in monetary terms.² However, for all operating lease

¹However, if a lease transfers ownership or includes a bargain purchase option, the term should be the leased asset's useful life instead of the lease term with appropriate consideration given to any residual value.

²With one exception. If there is a prepayment, the lessee must create a "Leasehold" and the lessor an "Unearned Revenue" account which is amortized in a reasonable and systematic fashion over the term of the lease.

agreements which exceed one year, the relevant terms and conditions of the lease must be disclosed in a footnote or on a supporting document.

In addition to the payment terms, the lease agreement may include an option to purchase or renew the lease. The agreement will also stipulate who is responsible for maintenance and repairs. More importantly, the agreement will identify which party is entitled to any investment tax credit. Typically, the lessor will retain the right to claim the investment tax credit. However, a noncorporate lessor may not be able to meet the necessary qualifications and be willing to pass the credit on to the lessee. The party entitled to claim the credit is typically not of importance because the terms of the lease will be adjusted accordingly. If the lessor retains the right to the tax credit, the lease will be negotiated at more favorable terms than if the lessor passes the credit to the lessee. Therefore, for the purposes here, it will be assumed the lessor retains the tax credit. The lessee is entitled to deduct the annual lease payments for tax purposes provided they are reasonable.¹

To evaluate the financial consequences of the lease, it is necessary to determine whether the analysis should be lease-or-buy or lease-or-borrow. Johnson and Lewellyn (67, pp. 815-824) argue the appropriate analysis is lease-or-buy whereas Van Horne (128, pp. 544-545) and Bower (19, pp. 24-34) argue that the question of profitability should be decided first (should the investment be made) and the question of

¹The Internal Revenue Service's primary concern is that the lease does not represent an installment sale.

financing (lease or debt) afterwards. Although this is a moot issue, the latter approach will be followed and profitability will be separated from financing, but the interrelationship will be kept in mind. The decision, in isolation, can be viewed in terms of the opportunity cost of funds and the time pattern of cash flows. Although a variety of techniques have been employed in evaluating the lease-or-borrow issue, only the internal rate of return method proposed by Beechy (10), Doenges (29), Findlay (39), Mitchell (94), Roenfeldt and Osteryoung (106), Wyman (135), and Long (80) will be presented here. For alternative approaches see Bower, Herrington and Williamson (20), Bower (19, p. 31), and Gordon (47). The alternative approaches utilize a net present value approach, the results of which are very sensitive to the discount rate incorporated in the analysis. Some authors employ the after tax cost of debt while others use the weighted average cost of capital. Since very seldom can 100 percent debt financing be employed, the first is inappropriate because it doesn't give recognition to equity financing. The problem with the cost of capital as a measure of the discount rate is that the cost of capital itself is determined by the decision to lease-or-borrow. Therefore, to avoid the problem of specifying a discount rate, the internal rate of return approach will be presented here. The internal rate of return, by making the rate endogenous, avoids the problem of having to specify the appropriate discount rate ex-ante.¹

¹However, even with the internal rate of return method, a cut-off rate below which the firm is not willing to invest must still be specified.

For a lease with no residual value and under the assumption that the lease payments equal the amortized payment plus imputed interest, the after tax cost of leasing can be determined by solving the following for I:

$$(A_0 - \text{ITC}) - \sum_{t=0}^{n-1} \frac{L_t}{(1+I)^t} + \sum_{t=1}^n \frac{T(L_{t-1} - P_t)}{(1+I)^t} = 0, \quad (4-29)$$

where

A_0 = cost of the asset to be leased,

ITC = investment tax credit foregone by the lessee,

n = number of periods of the lease,

L_t = lease payment in year t ,

T = marginal tax rate, and

P_t = depreciation charge in year t foregone by leasing, instead of owning.

The first term is the net after tax cost of the asset if it were purchased. The second term is the present value of the stream of lease payments discounted at the internal rate of return. The third term is the net tax savings (or tax dissavings) from deducting the lease payment instead of depreciation. The value of I which satisfies (4-29) is compared with the after tax cost of borrowing, or $r(1 - T)$. If $I < r(1 - T)$ then it is cheaper to lease. If $I > r(1 - T)$, it is cheaper to borrow.

Bonds

A bond can be defined as a contractual representation that a debt is owed by one party, the issuer, to one or more other parties, the investors. The indenture certificate indicates the principal amount, the stated interest rate based on the principal amount, and any other special agreements (131, p. 684).

The investor exchanges cash for the bond certificate, as shown in Figure 24. The issuer of the bond uses the cash proceeds to purchase productive assets which are employed in the production process. The cash generated from the sale of commodities is used to make the periodic interest payments and the principal payment at maturity required by the indenture. Any residual cash flow is available to the issuer or any shortage of cash flow must be made up from other sources.

The principal amount, the stated interest rate based on the principal amount, and the maturity date of the bond are all stated in the certificate. The effective rate of interest is determined by the price at which the certificate sells. If a bond sells for less than its face value, the bond is said to sell at a discount and the effective rate of interest on the bond is higher than its stated rate. Conversely, if the bond is sold for a premium (sales price exceeds face value), the effective rate is less than the stated rate. For large, publicly traded firms operating in a relatively efficient primary market, the firm selects the stated interest rate while the market determines the effective rate or the premium (discount) at which the bond will sell.

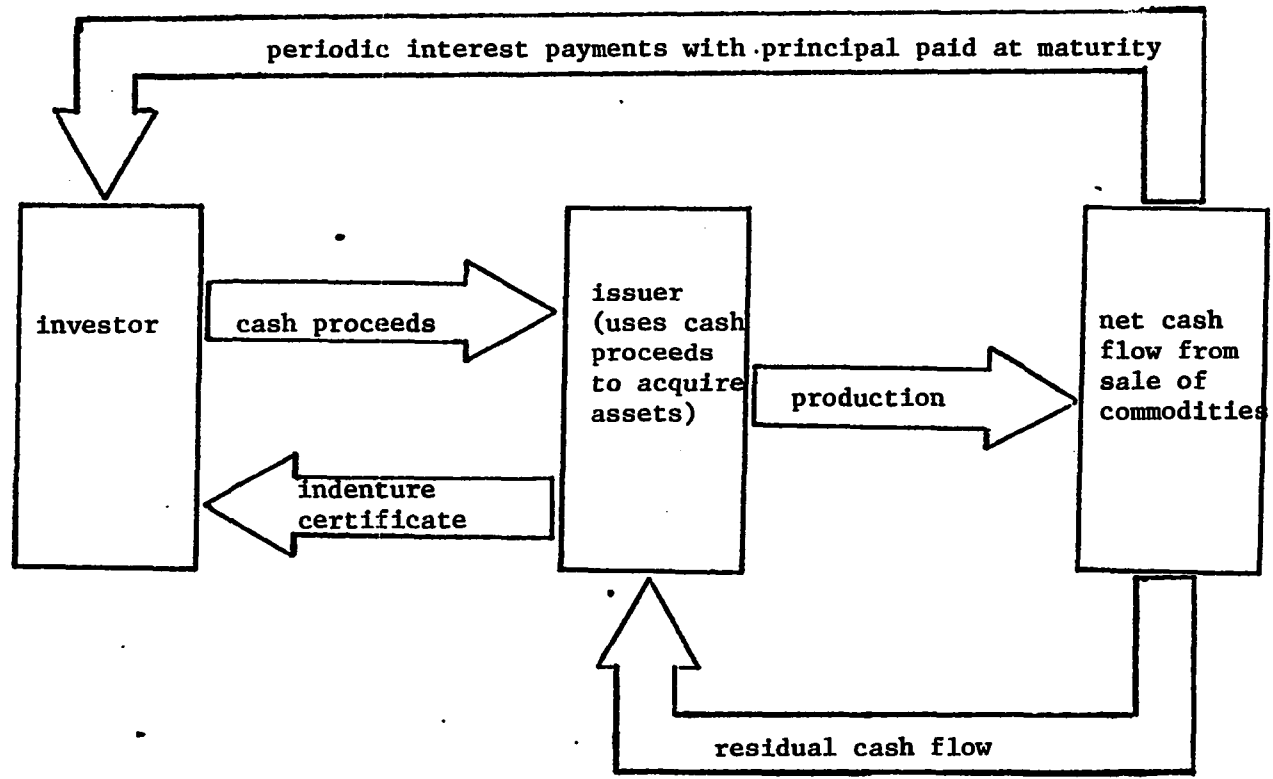


Figure 24. Bonds

The effect on the borrower's (issuer's) balance sheet of a bond issue is shown in Figure 25. The asset side is increased by the net cash proceeds received. The liability side is increased by the present value of the stated interest payments and principal payment at maturity discounted at the stated rate of interest. If the effective rate exceeds the stated rate, a negative discount entry must be made. If the effective rate is less than the stated rate, a positive premium account is entered. The cash proceeds will equal the par value minus any discount or plus any premium.

At the end of each interest period, an interest expense is charged to the income statement. Any discount or premium must be amortized over the life of the bond. The per period reduction in discount or premium is the difference between interest expense at the effective rate and interest expense at the stated rate. At maturity, the par value entry will be offset by an equivalent decrease in current assets and the discount or premium will be fully amortized. The firm pays explicit interest based on the principal amount but pays implicit interest determined by the effective interest rate.

In a family farm context, the parents and/or heirs may invest in bonds of the farm business at an interest rate different from a market determined rate. For example, the parents may pay the face amount for a bond with a stated rate of interest several percentage points below the market rate of equivalent securities. In this example, the parents have

Assets	Liabilities
+ cash proceeds	+ present value of interest and principal payments discounted at stated rate of interest + premium if effective rate is less than stated rate or - discount if effective rate exceeds stated rate

Figure 25. Effect of issuing a bond on issuer's balance sheet

given the firm preferential financing terms. The parents forego personal interest income but since they are also equity owners of the firm, they will participate in the increase in value in their residual ownership of the firm due to the preferential financing. In addition, at the preferential interest rate, the firm is subject to a lower degree of financial risk so that a trade-off exists between return and risk.

Installment contracts

Over one-third of all farm real estate debt in the United States is provided by individuals (8, p. 80 and 75, p. 208). They become lenders when they sell their land on contract or title transfer secured by a real estate mortgage. Most of these transfers are financed with an installment land contract.

Since the contract results from the direct negotiation of two parties who are usually closely related, the terms of the contract can vary greatly. The seller of the land passes control to the buyer along with the right to use the land. Usually, the title transfers at a later date. In exchange for this, the seller typically receives a small downpayment plus a written contractual obligation to pay interest, principal, and a balloon payment at stated dates. As shown in Figure 26, the buyer incorporates the newly acquired land in his production process. Cash generated from the sale of commodities is used to meet the debt servicing and principal repayments required by the contract. Any residual cash flow accrues to the buyer or must be made up from other sources if there is a shortage.

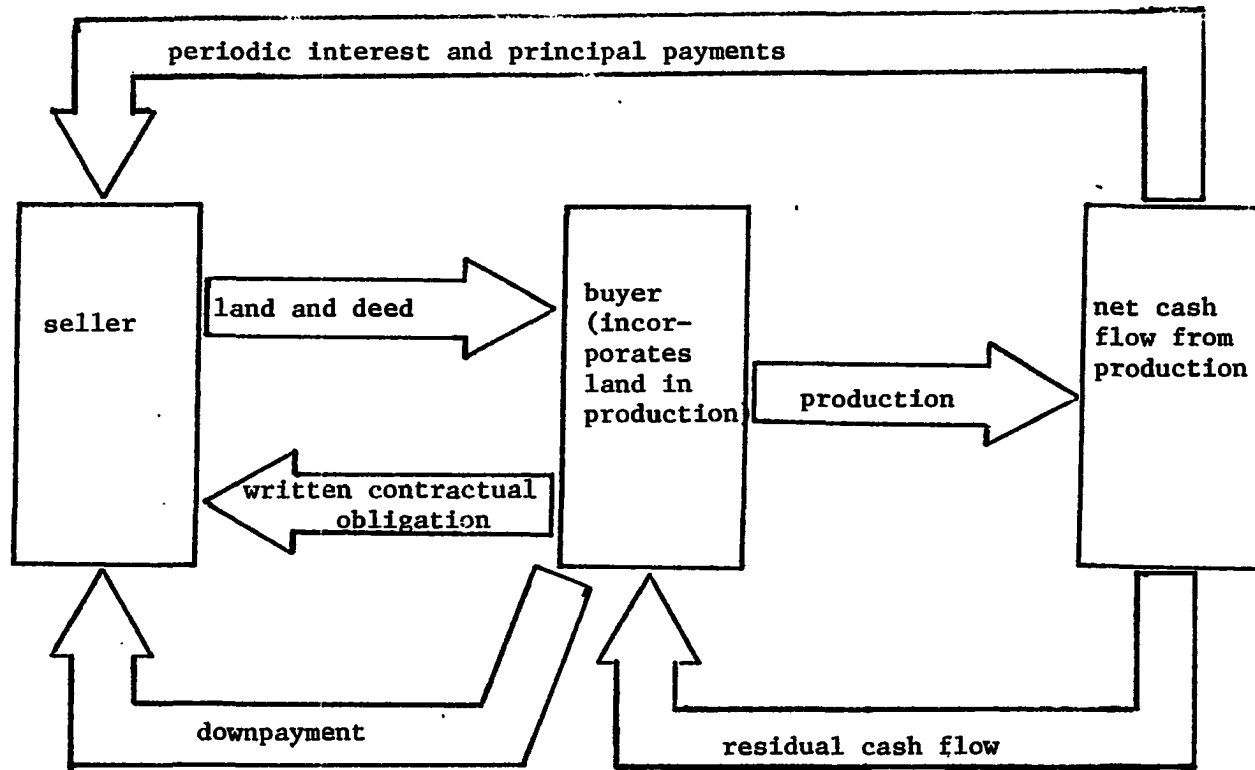


Figure 26. Installment sale land contract

The discounted sales price of an installment land contract is the present value of the downpayment, periodic principal and interest payments, and balloon payment (if any) discounted at the market rate of interest or

$$DSP = DOWN + \sum_{t=0}^{n-1} \frac{PR_t + INT_t}{(1+r)^t} + \frac{BAL + INT_n}{(1+r)^n}, \quad (4-30)$$

where ,

DSP is the discounted sales price,

DOWN is the downpayment,

PR_t is the principal payment due in year t,

INT_t is the interest payment due in year t,

BAL is the balloon payment,

n is life of the contract, and

r is the market rate of interest at year of sale.

The discounted sales price may not equal the fair market value of the land. Any discrepancy between the two is defined as the value of the contract, or

$$VC = FMV - DSP, \quad (4-31)$$

where

VC is the value of the contract,

FMV is the fair market value of the land, and

DSP is the discounted sales price.

The buyer's balance sheet would appear as in Figure 27 after the acquisition of the land on contract. The asset side is increased by the fair market value of the land. If this value differs from the discounted sales price, a contra-account equal to the value of the contract would also appear as a liability in addition to the discounted sales price net of the downpayment.¹ The value of the contract and the discounted sales price net of downpayment would be amortized annually as interest and principal payments are made.

The advantages to the buyer in an installment land contract are that the buyer can acquire a relatively expensive input for a small downpayment. The land can then be placed in production, and the net cash flows from the land's income generating ability can be used to meet the debt servicing needs of the contract. That is, the contract is self-liquidating. Furthermore, the interest rate and payment schedule are negotiable and can be tailored to fit the needs of the buyer. The contract is also advantageous to the seller in that it provides a predictable future stream of income. If the contract qualifies for installment reporting of gain, the seller can defer paying capital gains tax on appreciated land until the payments are received. That is, a portion of each payment is the recapture of basis (which is not taxable)

¹This is a departure from generally accepted accounting procedures which dictate the valuation of assets at the lower of cost or market. However, since land is typically the largest single entry on a land-extensive farming operation, valuation at cost may result in a substantial misrepresentation of asset values.

Assets	Liabilities
+ fair market value of land - cash downpayment	+ present value of periodic principal and interest payments discounted at the market rate of interest + value of contract

Figure 27. Effect of installment sales contract on buyer's balance sheet

and a portion which is capital gain and subject to tax. The interest payments received are taxed as ordinary income. Furthermore, in the event of forfeiture on the contract, it is relatively easy in Iowa for the seller to regain control of the land.

The next chapter will incorporate the materials of the last three chapters in the development of the empirical model.

CHAPTER V. DEVELOPMENT OF THE EMPIRICAL MODEL
AND DATA REQUIREMENTS

The purpose of model building is to develop a laboratory analogy of the system or process under study. For the problem to remain manageable, the analogue must abstract from reality but still remain sufficiently detailed to be useful in confronting theory with data (14, p. 43). The Introduction discussed the reasons why risk should be recognized in the decision making process. Chapter II developed the best criterion for incorporating risk in the decision making process. This "best" criterion is the maximization of expected utility measured in terms of net terminal wealth. Chapter III presented the use of first, second and third degree stochastic dominance as the method for measuring expected utility. Chapter IV discussed the theory of the firm from a whole farm perspective and discussed various methods of financing. This chapter presents the development of the empirical model and discusses the data requirements.

The first section of this chapter presents the empirical model used in this study. The next section describes the Monte Carlo technique used and the mechanics of implementing the results generated by the model into a stochastic dominance framework. Because taxes and the legal form of business organization play a very important role in the growth of the firm, the third section of this chapter develops the reasons for using the regularly taxed corporation as the legal entity for the family farm. The final section presents the data requirements and statistical parameters needed to run the empirical model.

The Empirical Model

Mathematical programming techniques such as linear and quadratic programming are widely employed to analyze decision problems. But the results generated by these or equivalent methods are only useful when the problem statement conforms with the rigorous parameter specifications required by the models. Alternatively, simulation provides much greater flexibility in modeling accounting, behavioral, and statistical relationships. If the underlying structural relationships and parameters are accurately specified, an appropriate search procedure will produce optimal (or at least near-optimal) results. It is the flexibility with which a simulation model can be adapted to a particular problem which constitutes its greatest strength. This is the primary reason why simulation is used in this study instead of a more rigid mathematical programming technique.

The Iowa State University Business and Financial Planning Model is used in this study.¹ The simulation model is an integrated collection of accounting identities, behavioral relationships, and stochastic variables which describe the growth or decay in the net terminal wealth of the family farm. As was shown in Figure 12, each family member owns some business assets and some personal assets. The business assets are transferred to the farm corporation in exchange for shares of common stock, certificates of indebtedness (term loans or bonds), leases,

¹The model is described in detail in Appendix C.

contracts, or some combination of the above. At the end of the planning horizon, each family member's wealth includes his (her) personal assets plus his (her) investor and equity-owner interests in the farm corporation.¹

The analysis provides a comparative static framework within which the effects of selected financing arrangements can be evaluated. Because the analysis is performed from a whole farm planning perspective, full recognition can be given to the effects of price and yield variability, inflation, and financing arrangements on cash flow streams which evidence less than perfect correlation over time. The desirability of a particular financing instrument or combination of instruments can be evaluated in terms of its effect upon the probability distribution of terminal wealth.

Throughout the presentation of the simulation model which follows, certain subscripts and superscripts carry a particular denotation. Unless otherwise indicated, these indices are:

Subscript t refers to the year of the analysis where $t = 1, 2, \dots$
 T (T is the last year of the planning period).

¹The farm corporation is assumed to be privately held and of such small size that its stock and bonds are not publicly traded on secondary markets. Stock values and financial instrument values will be determined by the fair market value of the assets against which they are written.

Sub- or superscript j refers to the j th family member, or to the corporation, i.e., $j = 0$ refers to the corporation, $j = 1$ is the father, $j = 2$ is the mother, $j = 3$ is the first on-farm heir and so forth.

Subscript k refers to the k th financial liability ($k = 1, 2, 3, \dots, K$) where financial liabilities include conventional term loans, intrafamily loans, bonds, leases, and sales contracts.

The equations are presented in relation to the accounting statement with which they are associated. The first set of equations calculates the after-tax income of the farm entity. The second set of equations describes the sources and applications of funds and the changes in financial position of the corporation. The third set of equations determines the balance sheet at the beginning of the subsequent period based upon the balance sheet of the previous period and the statement of changes in financial position. The last set of equations calculates each family member's cash flow and each family member's net worth at the end of the period.

Farm income

Equations 5-1 through 5-9 calculate the stochastic net income after taxes of the farm corporation for each year. Net operating income is defined as the difference between total revenue and variable operating costs. Mathematically

$$NOI_t = TR_t - VOC_t, \quad (5-1)$$

where

NOI_t is the net operating income in year t ,

TR_t is total revenue in year t , and

VOC_t is variable operating costs in year t .¹

As an alternative to calculating directly the two values on the right hand side of Equation (5-1), net operating income is empirically estimated in Equation (5-2) as a function of the amount of assets employed in the production process. The statistical coefficients in Equation (5-2) are empirically estimated from a pooled cross-sectional, time-series of Iowa Farm Business Annual Surveys for five sizes of representative farms (63). The estimated coefficients and relevant statistical values for each regression are presented in Appendix B. The net operating income in year t for a farm of size i is estimated as

$$\hat{NOI}_{it} = \hat{\alpha}_{0i} + \hat{\alpha}_{1i} \cdot CA_{it} + \hat{\alpha}_{2i} \cdot IA_{it} + \hat{\alpha}_{3i} \cdot FA_{it} + \hat{\alpha}_{4i} \cdot YR + \epsilon_{it}, \quad (5-2)$$

where

$\hat{\alpha}_{hi}$ are the estimated regression coefficients for a farming operation falling within size i as defined in Appendix B, Table B-2,

CA_{it} is the dollar amount of current assets as of the beginning of the production period of year t . Current assets are defined to have useful lives of less than 1 year,

¹Variable operating costs do not include operator labor. For convenience, operator labor is subtracted later.

IA_{it} is the dollar amount of intermediate assets as of the beginning of year t . Intermediate assets are defined to have useful lives of more than 1 but less than 7 years.

FA_{it} is the dollar amount of fixed assets as of the beginning of year t . Fixed assets have useful lives in excess of 7 years.

YR is the year of the analysis, and

ε_{it} is the error term which is assumed to be zero ($E(\varepsilon_t) = 0$).

Since total revenue is equal to prices of outputs times their respective quantities, and variable operating costs are equal to the input prices times the quantities used, Equations (5-1) and (5-2) implicitly measure, in dollar terms, the technological specification of the production processes employed. The inclusion of the year variable in Equation (5-2) captures the rate at which technological transformation takes place over time.

The predicted value of net operating income in Equation (5-2) is the expected value, that is, the value most likely to occur during a "normal" year. In reality, the value of net operating income actually realized may vary from the expected value. As discussed in Chapter 4 on the theory of the firm, this deviation from expectation is attributable to variability in prices and quantities paid and received. For the most part, this variability in net operating income is uncontrollable by the firm. Therefore, in Equation (5-3), the net operating income of Equation (5-2) is scaled by a Monte Carlo variate, x , which represents the

probability that the actual net operating income realized in year t may vary from its predicted value.¹ That is

$$NOI_t = \hat{NOI}_t \cdot x, \quad (5-3)$$

where

NOI_t is the actual net operating income realized in year t ,

\hat{NOI}_t is the predicted value from Equation (5-2), and

x is the Monte Carlo variate.²

If x equals one then the expectations are realized. If x is less than one, then the actual income realized is less than the expected: if x exceeds one, the converse is true.

Fixed operating costs are costs which are incurred regardless of the level of production. These costs are primarily due to the presence of assets which are not fully consumed within one production cycle. The major noncash fixed operating cost in agriculture is depreciation charges against machinery, equipment, and physical structures (i.e., assets which have intermediate and fixed lives). Cash fixed operating costs typically include such things as insurance, prepaid expenses, and maintenance and repair contracts which are amortized over the life of the asset. From the Iowa Farm Business Association Annual Surveys (63), cash and noncash

¹The development of the Monte Carlo variate, x , and the data used to generate it is deferred to a later section on Monte Carlo simulation.

²The subscript i (denoting representative farm size) is dropped through the remainder of the exposition unless needed for clarity.

fixed operating costs are regressed against intermediate and fixed assets in Equations 5-4 and 5-5. That is,

$$\hat{CFO}_{it} = \hat{\beta}_{oi} + \hat{\beta}_{li} \cdot IA_{it} + \hat{\beta}_{2i} \cdot FA_{it} + \hat{\beta}_{3i} \cdot YR + \emptyset_{it} \quad (5-4)$$

$$\hat{NCF}_{it} = \hat{\gamma}_{oi} + \hat{\gamma}_{li} \cdot IA_{it} + \hat{\gamma}_{2i} \cdot FA_{it} + \hat{\gamma}_{3i} \cdot YR + \psi_{it} \quad (5-5)$$

where

\hat{CFO}_{it} is cash fixed operating costs in year t,

\hat{NCF}_{it} is noncash fixed operating costs,

$\hat{\beta}_{ni}$ are the regression coefficients for farms belonging to size i for estimating cash fixed operating costs,

$\hat{\gamma}_{ni}$ are the regression coefficients for farms belonging to size i for estimating noncash fixed operating costs,

\emptyset_{it} , and ψ_{it} are the respective error terms, both with expected values of zero.¹

It is interesting to note that intermediate and fixed assets appear in both the estimation of net operating income and the estimation of fixed operating costs. Therefore, the net contribution of an intermediate or fixed asset to earnings before interest and taxes must be determined in light of Equation (5-3), (5-4) and (5-5) in concert and not just Equation (5-3).² Since, by definition, fixed costs are assumed to

¹The regression coefficients by size of farm are presented in Appendix B, Tables B-3 and B-4.

²The net cash contribution to earnings before interest and taxes is determined from Equations 5-3 and 5-4 only.

be constant over any production cycle, the predicted values in Equations (5-4) and (5-5) are also assumed to be the actual values realized.

That is, $CFO = \hat{CFO}$ and $NCF = \hat{NCF}$.

Earnings before interest and taxes (EBIT) are equal to the stochastically determined net operating income of Equation (5-3) minus the cash and noncash fixed operating costs of Equations (5-4) and (5-5). This relationship is

$$EBIT_t = NOI_t - CFO_t - NCF_t. \quad (5-6)$$

Net income before taxes in Equation (5-7) is equal to earnings before interest and taxes from Equation (5-6) minus total interest charges on servicing all interest bearing liabilities owed by the corporation minus all salaries and director's fees paid or,

$$NIBT_t = EBIT_t - \sum_{k=1}^K IP_{kt} - \sum_{j=1}^J S \& DF_{jt}, \quad (5-7)$$

where

IP_{kt} is the annual interest payment on liability k due in year t ,

$S \& DF_{jt}$ is the salary and/or director's fee paid to family member j .¹

¹Salaries and directors' fees are tax deductible only if they meet the IRS tests of reasonableness. The tests are based upon the amount of investment and personal labor contributed to running the corporation. The reason salaries and director's fees were not subtracted earlier, such as in Equation (5-3), is because these items are often used as income sharing devices. That is, the family members first determine how much is available for distribution as salaries and director's fees and allocate in accordance with this value.

The annual interest payment (IP_{kt}) in Equation (5-7) is determined by the outstanding balance on loans, bonds, leases, and contracts owed by the corporation. Or

$$IP_{kt} = r_k \cdot DEBT_{kt}, \text{ for } k = 1 \text{ to } k, \quad (5-8)$$

where

r_k is the annual interest rate on liability k ,

$DEBT_{kt}$ is the outstanding balance of liability k in year t .

For leases, the entire lease payment is tax deductible if it meets the IRS tests of reasonableness. Therefore, the right hand side of Equation (5-8) is imputed to equal the annual lease payment.

Net income after taxes is equal to net income before taxes from Equation (5-7) minus corporate income taxes. Or

$$NI_t = NIBT_t - TAXC_t, \quad (5-9)$$

where

NI_t is net income after tax,

$NIBT_t$ is net income before tax, and

$TAXC_t$ is corporate (subchapter C) federal income tax determined under the appropriate sections of the Internal Revenue Code.¹

¹The corporate tax rates have been reduced on the first \$50,000 of taxable income since this study was completed. The rate on the first \$25,000 of taxable income has been reduced from 17 percent to 15 percent and the rate on the second \$25,000, from 20 percent to 18 percent (25A, p. 21).

In an accounting context, net income after taxes would be closed to a retained earnings account. The retained earnings account less the dividends paid in that year plus the adjustments for changes in the economic values for assets would show the increase or decrease (when compared to previous years) in residual owners' equity in the corporation. The retained earnings account would then be added to the paid-in capital (or contributed capital) account to determine the book value of net worth. If all assets and liabilities are correctly valued and adjusted to reflect their economic values, then the value of net worth can also be determined as the residual of total assets minus total liabilities.

Sources and applications of funds and
statement of changes in financial position

Equations (5-10) through (5-15) describe the sources and applications of funds by the corporation and the changes which take place in the asset and liability accounts during any year. The total sources of funds for the corporation are the sum of net income after taxes from Equation (5-9) plus noncash fixed operating costs from Equation (5-5) plus new borrowings by the corporation plus new equity investment in the corporation. Mathematically

$$TS_t = NI_t + NCF_t + NB_t + NE_t, \quad (5-10)$$

where

TS_t is the total sources of funds in year t ,

NB_t is new borrowings by the corporation, and

NE_t is new investment in the corporation.

New borrowings by the corporation are the sum of the proceeds from the sale of new bonds and loans. That is,

$$NB_t = \sum_{k=K+1}^{K'} DEBT_{kt}, \quad (5-11)$$

where the index begins with $K+1$ to indicate these are new bonds and loans.

New equity in the corporation is the sum of the proceeds from the sale of additional shares of common stock to each of the family members, times the per share price as shown below.

$$NE_t = \sum_{j=1}^J (SH_{jt} \cdot PS_t), \quad (5-12)$$

where

NE_t is new equity investment in year t ,

SH_{jt} is the number of shares acquired by member j in year t ,
and

PS_t is the price per share paid in year t by member j and is calculated in Equation (5-27).

The total applications of funds by the corporation are equal to the sum of the annual principal payments on all liabilities owed plus

dividends paid on common stock plus new assets purchased, or

$$TU_t = \sum_{k=1}^K PP_{kt} + \sum_{j=1}^J D_{jt} + P_t, \quad (5-13)$$

where

TU_t is total applications of funds in year t ,

PP_{kt} is the principal payment in year t on liability k ,

D_{jt} is the dividend payment to individual j in year t , and

P_t is the total purchases of new assets in year t .

Total principal payments in Equation (5-13) include loans, contracts and bonds (in the year of maturity). Interest and lease payments have already been accounted for in Equations (5-7) and (5-8).

If the total sources of funds from Equation (5-10) exceed the total applications of funds from Equation (5-13), the excess funds are presumed to be reinvested in the corporation by purchasing new assets. If total applications exceed total sources, new borrowings are increased to equate total sources with total applications. Therefore,

if $TS_t > TU_t$, then

$$P_t = TS_t - \sum_{k=1}^K PP_{kt} - \sum_{j=1}^J D_{jt}, \quad (5-14)$$

and purchases are determined both exogenously and endogenously.

Similarly,

if $TS_t < TU_t$, then

$$NB_t = TU_t - NI_t - NCF_t - NE_t, \quad (5-15)$$

and new borrowings are determined by both internal and external forces. If Equation (5-14) is executed, it is necessary to recompute Equation (5-13) based on the new value of purchases. Conversely, if Equation (5-15) is executed, then it is necessary to recompute Equation (5-10) with the new value for new borrowings. Either path will always result in the equalization of total sources with total applications.

Statement of financial position

The changes in financial position described by the sources and applications account are used to adjust the financial statement account balances from one period to the next. The asset accounts (Equations (5-16) through (5-21)), liability accounts (Equations (5-22) through (5-27)) and net worth accounts (Equations (5-28) and (5-29)) are discussed in this section.

In Equation (5-16), total assets equals the sum of current assets plus intermediate assets less the accumulated depreciation taken against intermediate assets plus fixed assets minus the accumulated depreciation taken against fixed assets, or

$$TA_t = CA_t + (IA_t - AD-IA_t) + (FA_t - AD-FA_t), \quad (5-16)$$

where

TA_t are total assets at the beginning of year t ,

CA_t are current assets,

$AD-IA_t$ is accumulated depreciation taken against intermediate assets, and

$AD-FA_t$ is accumulated depreciation taken against fixed assets.

Accumulated depreciation on intermediate assets (calculated in Equation (5-17)) and fixed assets (calculated in Equation (5-18)) are the sum of the total depreciation charges taken against intermediate and fixed assets for all years. The accumulated depreciation equations are

$$AD-IA_t = \sum_{h=0}^t \hat{NCF}_h \frac{IA_h}{IA_h + FA_h}, \text{ and} \quad (5-17)$$

$$AD-FA_t = \sum_{h=0}^t \hat{NCF}_h \frac{FA_h}{IA_h + FA_h}, \quad (5-18)$$

where

\hat{NCF}_h is calculated in Equation (5-5).

The asset account balances at the beginning of the subsequent period (time $t+1$) are determined by adjusting the beginning balances of the previous period for the inflation, depreciation, and net investment that have occurred during the year. To maintain the same technical relationship between inputs and outputs, the relative proportions of the asset composition are assumed to remain constant. This is the same as saying that production occurs with fixed factor proportions. No substitutability of inputs is permitted. Therefore, investment or disinvestment of assets occurs proportionately, which ensures the same mix of assets is retained over time.

Current assets at the beginning of the next period are equal to current assets at the beginning of the current period (at their inflated replacement value) plus the acquisition of new current assets or

$$CA_{t+1} = CA_t (1+INF_t) + (CA_t/TA_t) \cdot P_t, \quad (5-19)$$

where

INF_t is the exogenously determined inflation rate in year t,

TA_t is total assets in year t from Equation (5-16), and

P_t is purchases in year t from Equation (5-10).

Intermediate assets (gross intermediate assets less the accumulated depreciation from Equation (5-17)) at their inflated value plus new intermediate assets purchased equals the value of intermediate assets at the beginning of the next period, or

$$\begin{aligned} IA_{t+1} = & (IA_t - AD-IA_t) (1+INF_t) + AD-IA_t + \\ & (IA_t/TA_t) \cdot P_t, \end{aligned} \quad (5-20)$$

where inflation is assumed to affect the replacement value of existing assets but not the depreciation taken against them.

Fixed assets are primarily land and structures where land is typically the largest single dollar item on the asset side of the farm balance sheet. During the 1970s, land has appreciated in value at a rate higher than the rate of inflation on current and intermediate assets (13% versus 7.4%, 35). Therefore, fixed assets at the beginning of the next period are equal to fixed assets at the beginning of the previous period less the accumulated depreciation taken against structures times an adjustment for inflation and the excess or shortfall of land appreciation over the inflation rate, plus new purchases of fixed assets, or

$$FA_{t+1} = (FA_t - AD - FA_t) (1 + INF_t)(1 + APP_t) + (FA_t / TA_t) \cdot P_t, \quad (5-21)$$

where

INF_t is the rate of inflation in year t , and

APP_t is the rate of appreciation in land values in excess of (or below) the rate of inflation.

Total assets at the beginning of the subsequent period are calculated using Equation (5-16).

The liability and equity side of the balance sheet is composed of current liabilities, long-term liabilities, and common stock. Current liabilities include such items as trade credit, accounts payable, wages and taxes payable, and short-term borrowings which are directly tied to the level of production activity. When production activity reaches its seasonal high, so, typically, do the above items. Current liabilities also include the current portion of the long-term liabilities. As was discussed in Chapter 4 on the conceptual considerations in financing, current liabilities are typically subject to a limited degree of discretionary control by management. Current liabilities are determined by the level of production and, more importantly, by the amount of assets needed to facilitate production. Furthermore, current liabilities are directly tied to current assets through the concept of working capital. Therefore, current liabilities at the beginning of the subsequent period

are defined to equal the current liabilities of the previous period plus the addition in current liabilities needed to compensate for the increase in replacement cost of current assets. That is,

$$CL_{t+1} = CL_t + CA_t \cdot INF_t, \quad (5-22)$$

where

CL_t is current liabilities, and

$CA_t \cdot INF_t$ is the increase in replacement value of current assets from Equation (5-19).

The current portions of long-term liabilities and short-term borrowings are included in Equation (5-22).

The remainder of the liabilities not included in Equation (5-22) are categorized as intermediate- and long-term liabilities which, for convenience, are grouped together into long term. Long-term liabilities include conventional operating loans and term loans, family loans, leases, bonds, and installment sales contracts owed by the corporation to family members and nonfamily institutions.

Loans

In accordance with Generally Accepted Accounting Procedures (99, Opinion No. 21), loans are assumed to be reported on the liability side of the balance sheet at the discounted value of future principal and interest payments. The appropriate rate of discount to employ is the market rate of interest on similar obligations of equivalent risk and maturity at the date of inception. When the loan is taken out at the

going market rate of interest, this rate is also the one used to discount the repayment stream. In this instance, the present value of the loan obligation is equal to its face value. Only if the cash proceeds of the loan differ significantly from the face value will the present value of the future principal and interest payments differ from the face value.

The value of each loan outstanding in year t is the present value of the remaining principal and interest payments discounted at the market rate of interest prevalent at its inception, as shown in Equation (5-23). That is,

$$DEBT_{kt} = \sum_{h=t}^T \frac{PP_{kh} + IP_{kh}}{(1+r_{ko})^h} \quad (5-23)$$

where

$DEBT_{kt}$ is the present value of loan k in year t ,

PP_{kh} is the principal payment on loan k due in year h ,

IP_{kh} is the interest payment on loan k due in year h ,
and

r_{ko} is the prevalent interest rate on similar loans in the year of inception.

Leases

"A lease is an agreement conveying the right to use property . . . , usually for a stated period of time" (131, pp. 748-780). For accounting purposes (in accordance with Generally Accepted Accounting Procedures), leases are defined as either capital or operating. A capital lease must be capitalized on both the asset and liability sides of the balance

sheet. An operating lease is not capitalized, but merely disclosed as an item of information in a supporting schedule or footnote.

However, if the present value of the minimum lease payment stream exceeds the fair market value of the property, then the property is included in the balance sheet at its fair market value. The asset and liability entries for a capital lease are amortized annually over the term of the lease payments, not the depreciable or economic life of the asset. The method of amortization should be consistent with conventional depreciation methods employed by the firm for similar assets which are not leased. The present value of a capital lease is calculated using Equation (5-23) where the sum of principal and interest payments due in year t is interpreted as the lease payment and the discount rate is the lessee's incremental borrowing rate or lessor's implicit rate of return, whichever is lower.

Bonds

A bond is a contractual representation that a debt is owed by one party, the issuer, to one or more other parties, the investors. A bond certificate indicates the principal amount, stated interest rate based upon the principal amount, specified interest payment dates and any other special agreements between the parties. Thus, a bond is a written promise to pay a specified principal at a stated date and, in addition, periodic interest on the principal at a specified rate per period (131, p. 684). The bond obligation appears as a liability which is valued at

its present value at the time of issuance. If the market rate of interest at issuance is above (below) the stated rate, then a discount (premium) associated with the sale of the bond will also be recognized as a liability as an offset to the face value of the bond. The amount of the discount or premium is the difference between the cash proceeds from the sale of the bond and the bond's face value. The discount or premium is amortized over the life of the bond. A bond sold at par in an arm's length transaction will have neither a discount nor a premium. The net liability (face value adjusted for discount or premium) of a bond is

$$DEBT_{kt} = \sum_{h=t}^T \frac{IP_{kh}}{(1+r_{ko})^h} + \frac{PP_{kT}}{(1+r_{ko})^T}, \quad (5-24)$$

where

$DEBT_{kt}$ is the present value of bond k in year t ,

IP_{kh} is the periodic interest payment, and

PP_{kT} is the principal payment (i.e., face value) in the year of maturity, T .

Installment sales contracts

Installment sales of land have become a very popular vehicle for transferring the ownership of farmland. Due to the rapid rate of appreciation of farmland in recent years, many land owners face substantial capital gains tax upon sale. Installment reporting of gain allows the seller to spread the recognition of capital gains over the life of the contract. At the same time, the contract allows the buyer to acquire the use and subsequent ownership of the land for a relatively low

downpayment. Furthermore, since most installment sales contracts are negotiated directly between seller and buyer without the involvement of a financial intermediary, both the sales price and the interest rate are negotiable.

Each contract entered into by the corporation will be valued as a liability calculated as the present value of the future principal and interest payments due. The market rate of interest at the date of inception is the appropriate rate of discount to use, or

$$DEBT_{kt} = \sum_{h=t}^T \frac{PP_{kh} + IP_{kh}}{(1+r_{ko})^h} + \frac{PP_{kT}}{(1+r_{ko})^T}, \quad (5-25)$$

where

$DEBT_{kt}$ is the present value of installment sale contract k in year t ,

PP_{kh} is principal payment due in year h ,

IP_{kh} is interest due in year h , and

PP_{kT} is the balloon payment (if any) due at maturity.

Total long-term liabilities are the sum of the present values of all loans, leases, bonds, and installment sales contracts not included in short-term liabilities, or

$$LTL_t = \sum_{k=1}^K DEBT_{kt} \quad (5-26)$$

Owners' equity

The residual ownership in the farm corporation is denominated as the fair market value of the common stock. The per-share value of the common is calculated as the fair market value of net assets (total assets minus total liabilities) divided by the number of shares outstanding, or,

$$PS_t = (TA_t - CL_t - LTL_t) / \sum_{j=1}^J \sum_{h=0}^t SH_{jh}, \quad (5-27)$$

where

PS_t is the per share value of common stock in year t and is the same for all family members (this is the value used in Equation 5-12),

SH_{jh} is the number of shares acquired by individual j in year h , therefore

$\sum_{h=0}^t SH_{jh}$ is the total number of shares owned by individual j and

$\sum_{j=1}^J \sum_{h=0}^t SH_{jh}$ is the total number of shares outstanding.

The aggregate value of the common stock can also be determined from Equation (5-27) as the difference between total assets and total liabilities.

Equations (5-1) through (5-27) completely describe the flow of funds through the farm corporation from an accounting standpoint. It is now necessary to describe the cash flows of the family members.

Cash flows of family members

For the family members, of which there are assumed to be three, the father and mother are treated as a single decision making unit ($j=1, 2$). It is assumed the parents share income and expenses equally and file a joint income tax return. It is also assumed there is one farm heir ($j=3$), who also qualifies for joint filing.

Taxable income is equal to the sum of nonfarm income plus salaries and director's fees from the corporation plus the taxable portion of dividends received plus interest payments on liabilities owed them by the corporation plus 40 percent of any recognizable gain from contract sales. For each family member,

$$\begin{aligned}
 TI_{jt} &= S\&D F_{jt} + NFI_{jt} + (D_{jt} + \sum_{k=1}^k IP_{kt}^j - 200) + \\
 &\quad \sum_{k=1}^k CG_{kt}^j, \text{ and} \qquad (5-28) \\
 D_{jt} + \sum_{k=1}^K IP_{kt}^j - 200 &\geq 0.^1
 \end{aligned}$$

¹For the tax year 1981, the first \$200 (\$400 if filing a joint return) of interest and dividend income is tax deductible. The deduction, however, can not exceed the income received. For tax years beginning after 1981, there is no exclusion for interest received. The exclusion on a joint return is \$200 for qualifying dividends regardless of which spouse has legal title to the stock (25b, p. 246).

where

TI_{jt} is the taxable income of individual j in year t ,

$S\&DF_{jt}$ is salary and directors' fee paid to individual j from Equation 5-7,

NFI_{jt} is nonfarm income received by individual j in year t ,

D_{jt} is dividends received by individual j in year t , from Equation 5-13,

$\sum_{k=1}^K IP_{kt}^j$ is the total interest payments in year t owed to individual j , and

$\sum_{k=1}^K CG_{kt}^j$ is the capital gain recognized for tax purposes on installment sale contracts owed to individual j .¹

The cash flow of the parents is their taxable incomes from (5-28) minus personal income taxes, minus personal consumption expenditures from Equation (5-31), plus principal payments received on loans, bonds, and contracts. New loans and bonds for which the individual is the lender or investor from Equation (5-11) must be subtracted from the above, as must the funds used to acquire new shares of common stock from Equation (5-12). For the parents and the heir,

$$\sum_{j=1}^2 CF_{jt} = \sum_{j=1}^2 TI_{jt} - TAXP_{jt} - \sum_{j=1}^2 C_{jt} + \sum_{j=1}^2 \sum_{k=1}^K PP_{kt}^j -$$

¹Beginning with Equation (5-28) it is necessary to add a superscript to designate the subset of all liabilities which involve the particular individual as the lender, lessor, investor or seller. In the absence of the superscript, or if it is otherwise clear, all individuals are included.

$$\sum_{j=1}^2 \sum_{k=K+1}^{K'} \text{DEBT}_{kt}^j - \sum_{j=1}^2 \text{SH}_{jt} \cdot \text{PS}_t \text{ and} \quad (5-29)$$

$$\text{CF}_{3t} = \text{TI}_{3t} - \text{TAXP}_{3t} - C_{3t} + \sum_{k=1}^k \text{PP}_{kt}^3 - \sum_{k=K+1}^{K'} \text{DEBT}_{kt}^3 - \text{SH}_{3t} \cdot \text{PS}_t, \quad (5-30)$$

where

CF_{jt} is the cash flow of individual j in year t ,

TI_{jt} is taxable income from Equation (5-28),

TAXP_t is personal income taxes from the IRS Tax Tables,¹

C_{jt} is consumption expenditures from Equation (5-31),

PP_{kt}^j is the principal payment on liability k received by individual j in year t ,

$\sum_{k=K+1}^{K'} \text{DEBT}_{kt}^j$ are new liabilities where individual j is the lender of a loan or investor of a bond, and

$\text{SH}_{jt} \cdot \text{PS}_t$ are funds used to acquire new shares of common stock.

Personal consumption expenditures appearing in Equations (5-29) and (5-30) are calculated using the consumption equation from the Iowa State University Computer Assisted Estate Analysis model as

$$C_{jt} = -37,419 + 619 (t-1900) + .04(\text{TI}_{jt} - \text{TAXP}_{jt}), \quad (5-31)$$

¹The personal tax rate tables have changed since this study was completed. The tax rate tables effective for the 1982 tax year are presented in 25b, pp. 13-19.

where

C_{jt} is the personal consumption expenditures of individual j , and
 t is the year.

The personal (nonbusiness) assets of the next period are the personal assets of the previous period plus any residual cash flows from Equations (5-29) and (5-30), or

$$PA_{j,t+1} = PA_{jt} + CF_{jt}, \quad (5-32)$$

where

PA_{jt} are the personal assets of individual j and
 CF_{jt} is the cash flow of individual j .

For simplicity, only one category of personal assets is recognized. This category includes both durable and nondurable assets.

Therefore, each family member's personal wealth is the sum of his personal assets, investor (and/or lender, lessor and seller) interests in the corporation, and owner's equity in the corporation. That is,

$$PW_{jt} = PA_{jt} + \sum_{k=1}^{K'} DEBT_{kt}^j + SH_{jt} \cdot PS_t \quad (5-33)$$

where

PW_{jt} is the personal wealth of individual j in year t ,
 PA_{jt} is the personal assets from Equation 5-32,

$\sum_{k=1}^{K'} \text{DEBT}_{kt}^j$ is the total present value of all old and new liabilities owed to individual j , and

$\text{SH}_{jt} \cdot \text{PS}_t$ is the fair market value of individual j 's equity interest in the farm corporation.

Equations (5-1) through (5-33) are repetitively solved for each year of the planning horizon. At the end of the planning horizon, the personal wealth for each family member from Equation (5-33) becomes one discrete point on the probability distribution of terminal wealth. The simulation is rerun with the same parameters (the Monte Carlo variate, x , remains stochastic) to generate a second observation of terminal wealth. This process is continued until the probability distribution is described to the desired degree of statistical reliability. This probability distribution is converted into a cumulative density distribution which constitutes the uncertain outcome of one risky financing arrangement. The cumulative frequency distribution for each alternative financing strategy is developed in a similar fashion. The cumulative density functions can then be compared using stochastic dominance theorems of Chapter III to determine which financing arrangement maximizes expected utility.

The Environment and Data Requirements

The empirical model presented in the previous sections implicitly assumes that the representative firm operates under certain environmental conditions. These environmental conditions are correlated cash flows and the corporate form of legal organization. This section will discuss these two conditions.

Correlation of cash flows over time

Net after-tax cash flow is the key determinant of the rate at which a firm grows in real terms.¹ In any year, the larger the ending cash-flow, the larger can be the investment in new productive assets to expand the production base for the subsequent period.

Most mathematical programming techniques and the majority of the researchers of firm growth assume the annual cash flows are either perfectly independent or perfectly correlated over time. This simplifying assumption greatly reduces the computational burden placed on the researcher and may be a reasonable approximation of reality when addressing questions of incremental investment analyses. However, when growth of the whole firm is modeled, either assumption may lead to unrealistic results. The reason for this has already been alluded to in terms of the linkage between the after-tax, after-dividends cash flow of any year and the resultant expansion or contraction of the production base for the subsequent period. An example will help clarify this concept.

Assume that a firm at time zero faces three possible outcomes in after-tax cash flows, as portrayed in Figure 28. If the firm experiences a very good year, it will follow the high cash-flow branch and have an after-tax cash flow of \$10,000. If the year is only average, then the

¹ Net after-tax cash flow is also after cash dividend payments have been made.

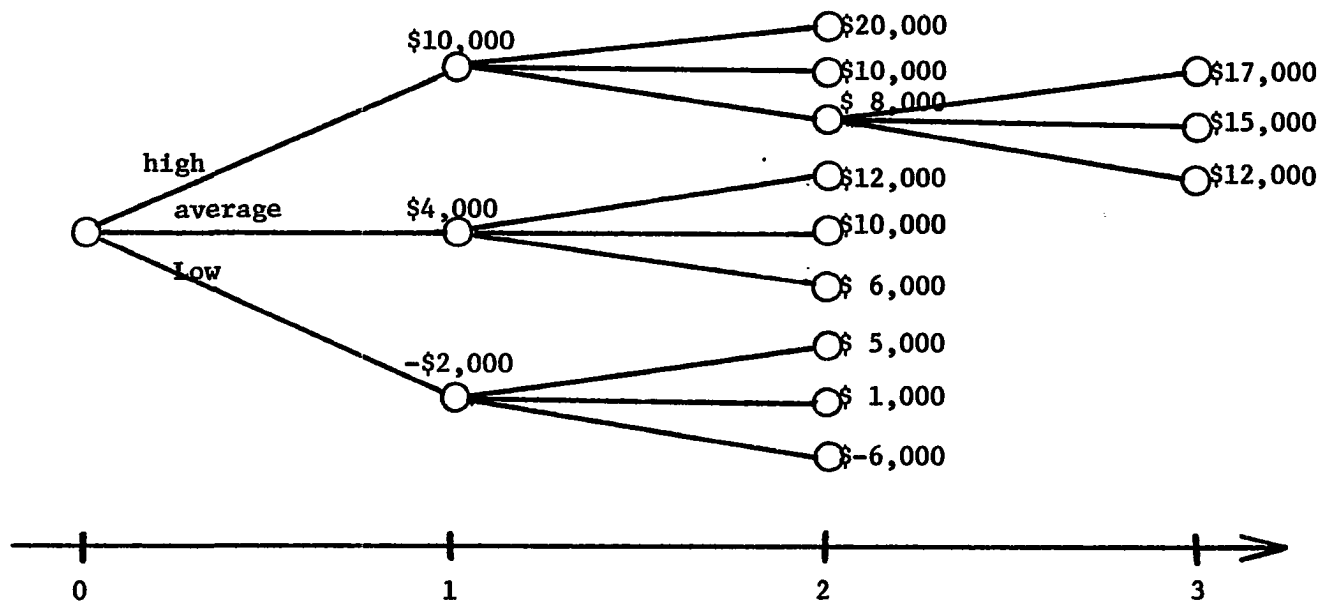


Figure 28. Decision tree for less than perfectly correlated cash flows

firm expects an after-tax cash flow of \$4,000. If the year is bad, the firm will follow the low branch and expect a negative cash flow of -\$2,000. But experiencing a bad first year does not entail a bad second year. That is, if the cash flow in year 1 is -\$2,000 it does not imply the expected outcome of year 2 will be -\$6,000. Nor does an above-average year in year 1 lead to an above average year 2. But this is exactly what would happen under the assumption of perfectly correlated cash flows. Under the assumption of perfectly correlated cash flows, the decision tree in Figure 28 would have only three, instead of nine, nodes at the end of year two. If the high branch is followed, then the expected cash flow at the end of year one would be \$10,000 and the expected cash flow at the end of year two would be \$20,000. If the average branch is followed, the expected cash flows would be \$4,000 and \$10,000. Similarly, the low branch generates cash flows of -\$2,000 in year one and -\$6,000 in year two.

Independent cash-flows imply that the outcome in year two is in no way determined by what happened in year one. In the example of Figure 28, each year would have the three outcomes of \$10,000, \$4,000 and -\$2,000.

In reality, the fact that year one turns out to be an above average year does not imply that year two will be also. Year two may be average or below average. But if year one is above average, then the larger cash flow can be used to increase the productive base going into year two. With constant scale considerations, increasing the production base in

year one increases the cash flow generating ability of the firm in year two. As a result, if the firm experiences another above-average year after an above-average year one, the firm's expected cash flow stream is \$10,000 in year one and \$20,000 in year two. If the firm experiences an above-average year after an average year, its expected cash flow stream is \$4,000 in year one and only \$12,000 in year two. Less than perfectly correlated cash flows imply that the outcome of one year determines the starting point (but not directly the ending point) of the next.

With the assumption of less than perfectly correlated cash flows, there are three possible outcomes after year one, nine outcomes after year two, 27 outcomes after year three, and so forth.

Mathematically, the relationship of these three assumptions can be described in terms of net present values and standard deviations of net present values. Under all three assumptions, the firm's expected net present value is

$$E(\text{NPV}) = \sum_{t=0}^n \frac{E(\text{CF}_t)}{(1+i)^t}, \quad (5-34)$$

where

$E(\text{NPV})$ is the expected net present value,

$E(\text{CF}_t)$ is the expected cash flow (after taxes and dividends) in year t , and

i is the risk free rate of interest which is assumed to remain constant over all n years.

Under the assumption of independence, the standard deviation of Equation (5-34) is

$$SD = \sqrt{\sum_{t=0}^n \frac{V(CF_t)}{[(1+i)^{2t}]}} \quad (5-35)$$

where

$V(CF_t)$ is the variance of cash flows in year t and is calculated as

$$V(CF_t) = \sum_{x=1}^H \left\{ CF_{xt} - E(CF_t) \right\}^2 P_{xt} \quad (5-36)$$

where

H is the number of discrete outcomes possible,

CF_{xt} is the cash flow of outcome x in year t ,

$E(CF_t)$ is the expected cash flow, and

P_{xt} is the probability of outcome x occurring.

Alternatively, if the cash flows are assumed to evidence perfect correlation over time, then the standard deviation is

$$SD = \sum_{t=0}^n \frac{SD_t}{(1+i)^t} \quad (5-37)$$

where

SD_t is the standard deviation in year t and is calculated as the square root of (5-36).

The standard deviation of (5-37) will always be larger than the standard deviation of (5-35). Because, for any series of positive numbers, the square of the sum is always greater than the sum of the squares. Or, in this case, the square root of the square of the sum (i.e., Equation (5-37)) will always be larger than the square root of the sum of the squares (i.e., Equation (5-36)).

If the cash flows are correlated over time (but not perfectly), then the standard deviation is calculated as

$$SD = \sqrt{\sum_{x=1}^m \left\{ NPV_x - E(NPV) \right\}^2 P_x}, \quad (5-38)$$

where

m is the total number of joint outcomes after n years,
or H^n ,

NPV_x is the net present value if joint outcome x occurs over
the n years,

$E(NPV)$ is from Equation (5-34), and

P_x is the joint probability of outcome x occurring.

Computationally, Equation (5-38) can become very tedious. If the example in Figure 28 with only three possible outcomes is carried out to ten years, 59,049 joint probabilities would need to be calculated. If the number of possible outcomes from any node is increased from three to five with a ten year planning horizon, then 9,765,625 joint probabilities would need to be calculated to generate one standard deviation. Although

problems of this magnitude are computationally possible, it should be evident why most researchers prefer to assume either perfect or zero correlation.

Alternatively, a Monte Carlo simulation can be used to approximate the final distribution of a correlated cash flow stream to any degree of accuracy desired.

Monte Carlo Simulation

In general, Monte Carlo methods are procedures which enable the researcher to set up a laboratory experiment of the real world within which the properties of the econometric estimators may be discerned (114, p. 1).

The steps involved in a Monte Carlo study are:

1. Specify a true structure of the problem being analyzed with exogenous and structural variables and parameters,
2. Generate a series of pseudo-random numbers from a preassigned distribution,
3. Solve the structure described in Step 1,
4. Repeat steps 2 and 3 for a number of samples, changing only the error terms, and
5. Evaluate the results.

The structure of the problem has been developed in Equations (5-1) through (5-33). The exogenous and structural variables and parameters will be presented later in this chapter and in Appendix B. The random number generation from a preassigned distribution is incorporated into

the simulation model in Equation (5-3) and the technique is presented in this section. The evaluation of the result will be done with stochastic dominance theory.

As was identified in the Chapter IV on the theory of the firm, there are three causes of risk to the firm. The first, production risk, is external and uncontrollable by the firm, and is manifested primarily through variability in prices and yields. The second cause, is investment or operating risk, due to the presence of fixed assets. The third cause, is financial risk, is attributable to fixed financing costs. The operating and financing components of total firm risk are handled endogenously within the structure of the simulation model once the initial set of parameters is established exogenously. It is the risk component caused by the variability in prices and quantities to which the Monte Carlo method is applied.

Step two of the Monte Carlo procedure requires the selection of a preassigned distribution from which the Monte Carlo variate can be randomly estimated. Many researchers have found the triangular probability distribution particularly useful in this regard (3, 28, 89, 100, 117). This is a popular and flexible distribution in eliciting personal preferences and in describing normal as well as nonnormal distributions. This is because the continuous triangular distribution can be completely described by three parameters--its lowest value,

highest value, and modal value. As pictured in the upper part of Figure 29, the horizontal axis measures the uncertain variate x and the vertical axis measures its probability of occurring. The point a is the lowest value which can occur, b is the largest value which can occur, and m is the mode or most likely value of x expected to occur. As shown in Figure 29, the distribution about the uncertain variate x is positively skewed but this need not be the case. If the mode is coincident with the mean and a and b fall three standard deviations below and above the mean, then the triangular distribution of Figure 29 approximates the normal distribution. All but one-half of one percent of the distribution of each tail is accounted for. Once a , m , and b are specified, the triangular probability distribution can be transformed into a cumulative density function, as shown in the lower half of Figure 29. It is from this cumulative density function that the pseudo-random numbers are generated in step two of the Monte Carlo procedure.

The pseudo-random number series is generated as follows. A random number between 0 and 1 is generated. This random number is the value u shown in the lower half of Figure 29. This value is mapped into the cumulative density function of x and the corresponding probability value γ of x is read from the horizontal axis. The value γ is used in Equation (5-3) as the error term to determine the actual value of net operating income realized in terms of its predicted value.

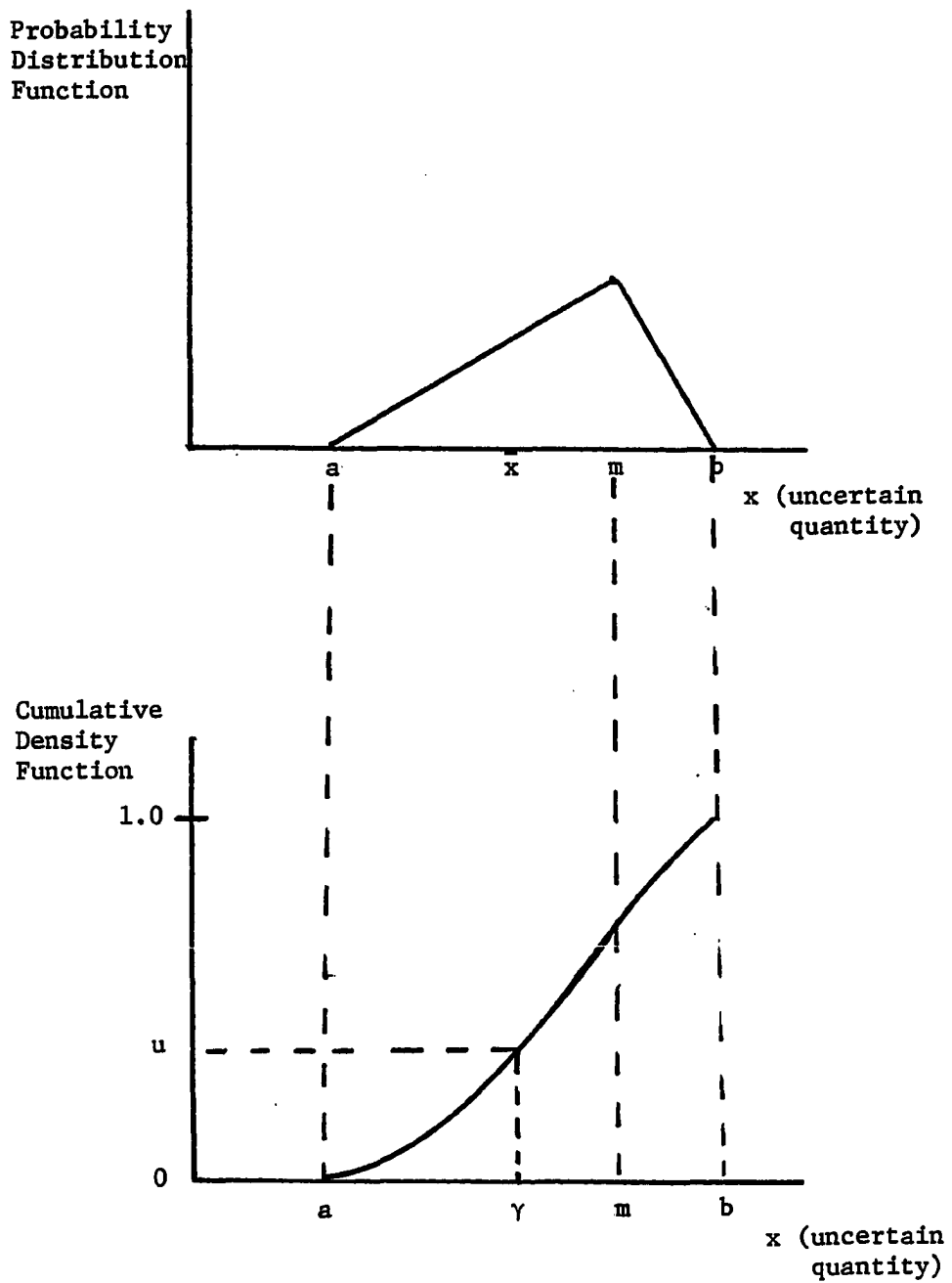


Figure 29. Monte Carlo simulation with a triangular distribution

Solving the cumulative density function of a triangular probability distribution for γ is a simple exercise in algebra.¹ The value for γ is

$$\begin{aligned}\gamma &= A + u(B-A)(M-A)^{1/2} \text{ if } u \leq (M-A)/(B-A), \text{ or} \\ \gamma &= B - (1-u)(B-A)(B-M)^{1/2} \text{ if } u > (M-A)/(B-A).\end{aligned}\quad (5-39)$$

where

A, M, B are the lower, modal and upper values of the triangular distribution, and

u is a random number between 0 and 1.

The uncertain variable x is a scalar where the mean is set equal to one and A, M, and B are stated in relation to the mean. The particular value, γ of x, represents the error term to be used in (5-3). If γ is equal to one, then the actual net operating income realized is the expected value by (5-3). If γ is less than one, then the actual is less than expected and conversely, if γ exceeds one, then the actual net operating income exceeds the expected value. The variate γ represents the inherent instability and variability in prices and yields. The particular values of the parameters A, M, and B are presented in Appendix B.

¹Such is not the case with many other distributions and in fact, no inverse exists for the normal and γ must be approximated.

One run of the simulation model for one selected set of parameters produces one point on the final probability distribution of net terminal wealth. The simulation is run again, allowing only the error terms in (5-3) to change, to generate a second point on the distribution. This process is continued until the probability distribution for net terminal wealth is approximated to the desired degree of accuracy. A cumulative density function can be created from the probability distribution function by assigning each discrete outcome the probability of $1/n$ for n runs. The solution values are ranked in ascending order and cumulatively summed to generate the cumulative density function of net terminal wealth for the chosen set of parameters. Choosing a different set of parameters will generate a new cumulative density function. The two sets of parameters can be evaluated in terms of the maximization of expected utility by comparing the two cumulative density functions with the stochastic dominance theorems of Chapter III.

The weakness of Monte Carlo modeling is that the optimum solution may never appear as one of the calculated solutions. However, Brooks (23, pp. 244-251 and 24, pp. 430-457) has statistically estimated the number of runs needed to obtain an optimal or near-optimal solution with a specified probability where the number of feasible, random observations required is

$$\pi = \log(1-P)/\log(1-\delta), \quad (5-40)$$

where

P is the probability that at least one observation will be made from the δ subset, and

δ is the proportion of the entire decision space which contains optimal or near optimal values of the decision variables.

The number of runs required for selected combinations of P and δ are shown in Table 5.

Referring to Table 5, if five percent of all possible solutions are assumed to be either optimal or near-optimal ($\delta=.05$) and the desired level of confidence is 90 percent ($P \geq .90$) that at least one of these solutions will be observed, it is necessary to run the simulation model forty-five times. That is, forty-five discrete points of the probability distribution of terminal wealth are sufficient to insure ninety percent confidence of including at least one optimal or near-optimal solution among the forty-five.

Corporate form of organization

As identified in (5-9), the farm firm is assumed to be a regularly taxed, Subchapter C corporation. Although most farms in the past and at present are sole proprietorships or spin-offs into informal partnerships, there is an increasing trend toward incorporation. In large part, this trend toward incorporation is attributable to income and estate tax considerations. The most important of these inducements to incorporate are:

Table 5. Number of feasible, random observations, Γ , required for selected values of ρ and δ

		ρ			
		.80	.90	.95	.99
δ	.05	32	45	59	90
	.025	64	91	119	182

1. Increased flexibility in estate planning and flexibility in the transfer of property prior to and after death.
2. Flexibility in integrating young family members into the farming operations.
3. Income-sharing plans which make use of both corporate and personal tax provisions to reduce federal income taxes.
4. Continuity of existence after the death of the major shareholder.
5. Liability of shareholders limited to the extent of their investment in the corporation.
6. Tax deductible fringe benefits such as retirement plans, life insurance premiums, and health insurance.

But the regularly taxed corporation also suffers a number of disadvantages over the sole proprietorship. These disadvantages include:

1. At least as high and usually higher tax rates on capital gains.
2. Higher costs of social security taxes, Worker's Compensation, and unemployment insurance.
3. The business activities of the corporation are limited to those specified in the articles of incorporation.
4. Some states restrict corporate activity in agriculture.
5. Corporations typically require more extensive recordkeeping.

In general, it is not clear whether the advantages outweigh the disadvantages. Each situation must be evaluated in light of its

particular circumstances and the objectives of the decision maker. In addition, the sole proprietorship or informal partnership can be structured in such a fashion as to function much the same as the corporation. Therefore, although the corporate form of organization is used in the simulation model of (5-1) through (5-33), the model could also be structured in terms of a sole proprietorship or informal partnership with much the same results.

Data Requirements

In an effort to keep the results of the simulation model of (5-1) through (5-33) as generally applicable as possible, balance sheets and income statements were developed for five representative Iowa farms based on the Iowa Farm Business Association's Annual Surveys (63). USDA publications (61, 62, 125, 126, 127, and 26) were used to estimate nonfarm income, nonfarm personal wealth, inflation, and market rates of interest.

Balance sheet

One of the classification schemes of the Iowa Farm Business Association is by acreage where there are five class sizes, as shown in Table 6. For each class, 1980 current, intermediate, and fixed asset categories were predicted using an autoregressive model with a one year lag of the form

$$Y_t = X_t \beta + \mu_t, \text{ and} \quad (5-41)$$

Table 6. Iowa Farm Business Association's size classification by acreage^a

Class	Acres
1	100 - 179
2	180 - 259
3	260 - 359
4	360 - 499
5	500 and over

^aFrom Iowa Farm Business Associations (63).

$$\mu_t = \rho\mu_{t-1} + e_t,$$

where

t is for the years 1970 through 1979, and the standard statistical properties are assumed to apply, namely

$$e_t \sim (0, \sigma^2),$$

$$e_t, e_j = 0 \text{ for } t \neq j,$$

$$|\rho| < 1,$$

$$E(u_t) = 0,$$

$$V(u_t) = \frac{\sigma^2}{1-\rho^2}, \text{ and}$$

$$\text{COV}(u_t, u_{t-h}) = \frac{\rho^h}{1-\rho^2} \sigma^2.$$

The dependent variable Y_t in (5-41) is the asset type being projected.

The independent vector of variables X_t are an intercept term and the year. The coefficients and statistical results are presented in Appendix B.

The 1980 projected assets for each size of farm of Table 2 are presented in Table 7. A class one farm has 155 total acres and 136 rotated acres. It has \$85,232 of current assets (such as feed, livestock, supplies, and stored grains); \$72,257 of intermediate assets (primarily machinery, breeder livestock, and equipment); and \$243,501 of fixed assets (land and structures) for a total of \$440,990 in business assets. A class two farm has \$117,028 of current, \$107,170 of intermediate, and \$341,892 of fixed assets for a total of \$566,090.

Table 7. Asset composition by size--1980 projections

Class size	1	2	3	4	5
Assets					
Current	\$ 85,232	\$117,028	\$186,810	\$218,647	\$ 377,350
Intermediate	72,257	107,170	139,573	147,930	247,142
Fixed	<u>243,501</u>	<u>341,892</u>	<u>455,125</u>	<u>597,621</u>	<u>1,014,168</u>
Total assets	\$400,990	\$566,090	\$781,508	\$964,198	\$1,638,660
Total acres	155	225	328	427	777
Rotated acres	136	196	282	359	614
Personal assets	\$ 8,020	\$ 11,322	\$ 15,630	\$ 19,284	\$ 32,773

Similarly, a class five farm with 777 total acres (614 are rotated cropland) has \$377,350 of current assets, \$247,142 of intermediate, and \$1,014,168 of fixed assets for a total of \$1,638,660 in 1980.

The Iowa Farm Business Association does not survey nonfarm, personal assets. Based on a U.S. Department of Agriculture study (125, pp. 33-36,) projected 1980 nonfarm personal assets are calculated as two percent of business assets. As shown in the lower part of Table 7, personal assets of class one farms are \$8,020 in 1980. Personal assets of a class three farm are \$15,630 and a class five has \$32,773 in personal assets.

Although the reporting of asset values in the Iowa Farm Business Associations surveys is required of all members and verified by enumerators, liabilities are an optional reporting item and are not verified. As a result of this nonreporting (and suspected under-reporting) of liabilities, the Iowa Farm Business Association's surveys had to be adjusted to determine the liabilities of the representative farms.¹

Liabilities of the representative farms were estimated as a composite of four sources of information. A personal interview Mr. Doug Meline, who is a Iowa Farm Business Association farm consultant and head enumerator (91), suggests the debt to asset ratios of the representative

¹Debt to asset ratios of the Iowa Farm Business Associations surveys varied from four percent for class one to seven percent for class five farms. Most researchers, including me, believe these values to be unrealistically low.

farms should range from twenty percent for class one to thirty percent for class five. The Balance Sheet of the Farming Sector (125, p. 39) indicates that the overall debt to asset ratio of Iowa farms is sixteen percent.

A disaggregated capital projections study done by Boehlje and Reinders 62, pp. 61-92) indicates the debt-to-asset ratios for cash grain farms are twelve to fifteen percent and livestock farms are sixteen to thirty-one percent. The findings of this study, the cross classification from the economic sales class to IFBA acreage classes, and the debt-to-asset ratios for cash grain and livestock are summarized in Table 8.

The Agricultural Finance Outlook (126, p. 26) suggests that the debt-to-asset ratios for U.S. livestock, dairy, and grain farms classified by small, medium, and large should vary from 8.1 percent to 30.4 percent, as summarized in Table 9.

Based on these four sources of information, the liabilities reported on the Iowa Farm Business Association's surveys were increased to the range of fourteen to twenty-two percent, as shown in Table 10.¹ For a class one farm, current liabilities are projected to be \$36,962 and noncurrent to be \$22,177, for a total of \$59,139 in 1980. Class three

¹Although the asset values are critical in estimating net operating income, the liability values serve only as a starting point of the analysis. The primary focus of this analysis is upon the effects of selected financing arrangements (presented in a later section of this chapter). Therefore, the liability projections presented in Table 10 merely provide a "reasonable" picture of the financial position of the representative farms as a base from which to begin the analysis.

Table 8. Projected 1980 debt-to-asset ratios for cash grain and livestock farms^a

Sales	IFBA Class ^b	-----Debt-to-asset ratio-----	
		Cash Grain	Livestock
III-V	1	12%	16%
II	2	13%	18%
IB	3, 4	14%	24%
IA	5	15%	31%

^aFrom Balance Sheet of the Farming Sector 1979: Supplement (125).

^bFrom Table 6.

Table 9. Debt-to-asset ratios by type and size^a

Size	Livestock	Dairy	Cash Grain
	-----Percent-----		
Small	8.1%	14.7%	13.3%
Medium	12.5%	25.7%	14.0%
Large	20.9%	30.4%	16.2%

^aFrom Agricultural Finance Outlook (126).

Table 10. Debt-to-asset ratios and liabilities by farm size--1980 projections

	Acreage Size ^a				
	1	2	3	4	5
Debt-to-asset ratio	.14	.16	.18	.20	.22
Liabilities					
Current	\$22,177	\$24,093	\$ 55,425	\$ 64,987	\$124,735
Noncurrent	<u>36,962</u>	<u>66,482</u>	<u>85,246</u>	<u>127,853</u>	<u>235,770</u>
Total Liabilities	\$59,139	\$90,575	\$140,671	\$192,840	\$360,505

^aSee Table 6.

farms are projected to have \$55,425 of current and \$85,246 of noncurrent liabilities in 1980 for a total of \$140,671. Class five farms are projected to have \$360,505 of total liabilities in 1980 composed of \$124,735 current and of \$235,770 noncurrent liabilities.

Farm income

From the Iowa Farm Business Association's surveys, farm income and expenses of the representative farms were separated into three components—net operating income, cash fixed operating expenses, and noncash fixed operating expenses. Net operating income, as defined in (5-1), is total revenues minus total variable operating expenses. Equation (5-2) was used to estimate net operating income as a function of the stock of existing assets, an intercept term, and the year for each representative farm. The model of (5-41) was used to estimate the starting asset values in (5-2). The coefficients, t-values, lag coefficient, and R^2 are summarized in Appendix B.

Cash fixed operating expenses of (5-4) and noncash fixed operating expenses of (5-5) were estimated for each class as a function of non-current assets, an intercept term, and time using the model in (5-41). The coefficients, t-values, and R^2 are presented in Appendix B.

Earnings before interest and taxes from (5-6) are the residual of net operating income and operating expenses. The 1980 projected earnings before interest and taxes and the percentage return on total assets are summarized in Table 11 for each representative farm.

Table 11. Projected 1980 earnings before interest and taxes by class

Class	Net operating income	Cash fixed expense	Noncash fixed expense	Earnings before interest and taxes	Percent return on total assets
1	\$ 38,634	\$ 4,374	\$ 9,582	\$ 24,678	6.2
2	\$ 66,550	\$ 5,784	\$11,897	\$ 48,869	8.6
3	\$ 67,467	\$ 5,931	\$14,110	\$ 47,426	6.1
4	\$ 86,253	\$ 8,695	\$13,175	\$ 64,383	6.7
5	\$148,321	\$13,675	\$24,993	\$109,653	6.7

From Table 11, a class two farm in 1980 is projected to generate a net operating income of \$66,550 in 1980, cash fixed expenses of \$5,784 and noncash fixed expenses of \$11,897 for a projected earnings before interest and taxes of \$48,869. This is an 8.6 percent return on total assets before taxes and interest. Similarly, a class five farm in 1980 is projected to earn \$109,653 before taxes and interest based on a net operating income of \$148,321, cash fixed expenses of \$13,675 and noncash fixed expense of \$24,993. The projected return on total assets for a class five farm is 6.7 percent.

Nonfarm income

Data available on nonfarm income are sketchy. The Iowa Farm Business Association's surveys concentrate entirely on farm and farm related sources and collect no information on nonfarm income.

John Crecink, in a U.S. Department of Agriculture study (26, pp. 32 and 45), estimated nonfarm income as a percentage of total family income for the United States, as shown in Table 12. For example, a farm family with a total income of \$4,000 would have earned \$1,600 from farming and \$2,400 from nonfarm sources. A family with a total family income of \$25,000 would have earned \$15,000 of that amount from farm sources and \$10,000 from nonfarm sources. Crecink found that the primary sources of the nonfarm income for corn belt farmers are nonfarm wages and salaries (50%), nonfarm business investments (30%), pensions (14%), and other investments (5%) (26, p. 45).

Table 12. Farm and nonfarm income as a percent of total family income in farming for given levels of income^a

Level of total family income	As a percent of the total	
	Farm	Nonfarm
\$ 2,000	20	80
4,000	40	60
10,000	35	65
23,000	50	50
25,000	60	40

^a Crecink, (26, p. 32).

Crecink's findings are supported by Farm Income Statistics (127, p. 59) which found that smaller farms rely more heavily on nonfarm sources of income while the larger farms generate the majority of their income from farm sources¹. In light of these two studies, and since the earnings levels of the representative farms all exceed \$20,000, it is assumed that for all but the largest farm size, farm income represents three-fourths of the total family income while nonfarm sources represent the remaining one-fourth. For the largest farm size, it is assumed that farm income represents four-fifths and nonfarm income represents one-fifth of total family income.

Market rate of interest and rate of inflation

For outstanding real estate and nonreal estate debts, market rates of interest of nine and twelve percent are used (126, pp. 12-13). The projections of future market interest rates and inflation rates used in this study are taken from the aggregate, baseline macro projections of the National Agricultural Credit Study (62, p. 64). These projections are summarized in Table 14.

This completes the development of the empirical model and the data employed. It is now appropriate to discuss the financing arrangements selected for analysis and the empirical findings.

¹The Farm Income Statistics figures for 1978 are summarized by economic sales class in Table 13.

Table 13. Net farm and off-farm income levels by volume of sales in 1978^a

	ALL	IV-V	III	II	IB	IA
Net farm income Before inventory adjustment	\$10,037	\$ 3,281	\$ 5,917	\$11,745	\$21,636	\$52,337
Off-farm income	12,829	13,573	10,068	7,802	6,846	10,850
Total income	\$22,866	\$16,854	\$15,985	\$19,547	\$28,482	\$63,187

^aFrom Stiglitz (119, p. 59).

Table 14. Projections of macroeconomic variables, 1980-1990^a

Year	1980	1981	1982	1983	1984
U.S. GNP (\$Bil.)	2,570	2,841	2,310	3,538.	3,914
percent change	8.6	10.5	13.0	10.2	10.6
Consumer price index (1967 = 100)	246.6	270.4	291.2	315.9	341.0
percent change	13.4	9.7	7.7	8.5	8.0
Prime interest rate	15.0	11.0	8.5	10.7	10.3
percent change	22.1	-26.6	-22.7	26.4	- 4.0

^aFrom 62.

1985	1986	1987	1988	1989	1990
4,310	4,732	5,194	5,688	6,221	6,791
10.1	9.8	9.8	9.5	9.4	9.2
376.6	395.2	423.9	454.0	485.3	518.1
7.8	7.5	7.3	7.1	6.9	6.7
10.3	10.4	10.6	11.1	11.6	12.2
- 0.2	1.0	2.3	4.2	4.6	5.3

CHAPTER VI. EMPIRICAL FINDINGS

Chapter I discussed the importance of financing in facilitating the intergenerational transfer of the family farm in an uncertain environment. Chapter II developed the maximization of expected utility measured as net terminal wealth as the best criterion for choosing between alternate financing methods. Chapter III developed stochastic dominance as the appropriate method of choosing the better of a pair of financing methods which results in the maximization of expected utility. Chapter IV discussed the role and conceptual considerations of financing as an integral part (along with production and investment) in the theory of the firm. Chapter V presented the empirical Monte Carlo simulation model and data needed to run the model. This chapter presents the financing methods selected for analysis and discusses the empirical findings.

Financing Situations Selected for Analysis

The family unit used in each financing situation is composed of a father and husband, age 55, a mother and wife, age 48 and a married heir, age 21. All are in excellent health and actively participate in the management and operation of the farm. The father and mother are assumed to function as a single decision making unit. They share income and expenses equally and file a joint federal income tax return. The heir is assumed to have his (her) own family so that he (she) also is entitled to

file joint federal income tax returns. The heir is an on-farm heir who will gradually take over management and ownership of the farm as the parents reduce their role in the farm. Ultimately, upon the parents' death, the heir will inherit full ownership of the farm. The financial situations selected for analysis address the issue of facilitating the intergenerational transfer process while the parents are still living and actively participating in management.

Table 15 summarizes the financial situations selected for analysis.¹ For each of the representative farms summarized in part A of Table 15, each family member receives an annual salary starting at \$2,200 for classes II, III and IV and \$1,100 for class V.² The

¹Class I farms were dropped from the analysis because these represent a large number of part-time and small farms incapable of sustaining a family as the primary form of income.

²These salary levels were chosen to approximately balance each individual's initial consumption needs with his or her initial after-tax income. It is possible to fault these salary levels as being too low by economic standards as a return on an input and low by tax standards (50b, Chapter 57). See *Bramlette Building Corp.* 52 T.C. 200 (1969), *aff'd*, 424 F.2d 751 (5th Cir., 1970); Gary N. Cromer, T.C. Memo. 1980-263; Pat Krahenbuhl, T.C. Memo. 1968-34; and Martin Fundenberger, T.C. Memo. 1980-113. If salaries are increased to a more reasonable level, corporate after-tax cash flow will decline by a factor of one minus the corporate marginal tax rate, times the increase in salaries, or the corporate after-tax cash flow will decrease by

$$(1 - \text{TAXC}) \cdot S$$

where TAXC is the marginal tax rate paid by the corporation, and S is the increase in salaries.

The increased salaries will increase the after-tax cash flow of each individual by

$$(1 - .04 - (1 - .04) \text{TAXP}) S$$

where .04 is the slope coefficient on after-tax income from the consumption function on page 155 and TAXP is the individual marginal tax rate.

The increased salaries reduce the amount of investment and new assets by the corporation through its retained earnings, but increase the

starting salary is increased annually by the projected rate of inflation (shown in Table 14, page 188). Nonfarm income for each family member is estimated as a percentage of net income. For classes II, III and IV, each family member is assumed to earn 8.33 percent of total income from nonfarm sources. Collectively, the family earns 25 percent of total income from nonfarm sources and 75 percent from farm sources. Restating nonfarm income as a percentage of farm income, nonfarm income is 1/3 for

Footnote continued from previous page
salaries would alter the quantitative results presented in the remainder of this chapter. The amount of alteration would depend on the values of TAXC, TAXP, S, and the ownership mix of common stock between the parents and the heir. For each of the situations analyzed, the marginal tax rates and the ownership mix of common stock change over the planning horizon. But using the averages as proxies for the marginal tax rates for the class IV farm, owned 80 percent by the parents as presented in Table 16, the effects of an increase in salaries can be analyzed in the following fashion. From Table 16, let TAXC = .18, TAXP = .15, and S = \$30,000 (an increase of \$10,000 per individual). The average decrease in the corporate after-tax cash flow would be

$$(1 - .18) \cdot \$30,000 = \$24,600.$$

The average increase in the after-tax cash flow of the parents will be

$$(1 - .04 - (1 - .04) \cdot .15) \cdot \$20,000 = \$16,320.$$

The average increase in the after-tax cash flow of the heir will be

$$(1 - .04 - (1 - .04) \cdot .15) \cdot \$10,000 = \$8,160.$$

Because the parents own 80 percent of the stock, the value of their common stock would be

$$-(\$24,600)(.80) + \$16,320 = -\$3,360.$$

The net effect for the heir would be

$$-(\$24,600)(.20) + \$8,160 = \$3,240.$$

Over the ten year planning horizon, the midpoint of the parents' net equity would be \$33,600, or about 2 percent smaller (\$1,907,462 versus \$1,941,062). The midpoint of the heir's net equity would increase by \$32,400, or by about 7 percent (\$517,665 versus \$485,265). When the heir converts half the common stock into a loan or a bond, the midpoint of the parents' net equity would decline by more than \$33,600 while the heir's net equity would increase by more than \$32,400. However, the relative ranking of preferences is not likely to be altered except in those situations where the curves are already so close that there is little significant difference between them.

Table 15. Financial situations selected for analysis

Farm Class	II	III	IV	V
A. Equities and Incomes				
Total equity	\$310,400	\$640,800	\$771,200	\$1,278,000
Salaries (each)	2,200	2,200	2,200	1,100
Nonfarm income as a percent of farm income (each)	.11	.11	11	8.33

B. Ownership Mix				
Parents	100%	80%	60%	
Son	0%	20%	40%	
	<hr/>	<hr/>	<hr/>	
Total	100%	100%	100%	

C. Parents; and Heir's Ownership/Investor Mix				
Common stock:	100%	50%	50%	
Loan:	0%	50%	0%	
Bond:	0%	0%	50%	
	<hr/>	<hr/>	<hr/>	
Total:	100%	100%	100%	

D. Interest rates 12% and 9%				

E. Dividends salaries with no dividends on common stock no salaries with dividends of 10 percent				

the family or 11.11 percent for each family member. For class V farms, the family is assumed to earn 20 percent of total income from nonfarm sources and 80 percent from farm sources. Assuming nonfarm income is earned equally, then each family member earns 8.33 percent in nonfarm income as a percentage of farm income.

Three ownership mixes are analyzed for each farm class, as summarized in Part B of Table 15. The first situation assumes that the parents own 100 percent of the farm and the heir owns nothing. The second situation is divided 80 percent to the parents and 20 percent to the heir. The third situation is divided 60 percent to the parents and 40 percent to the heir.

Part C of Table 15 summarizes the heir's and parents' options on their ownership mix. In the first situation of Part C, the heir owns his their entire interest (if any) in common stock. In the second situation of Part C, the heir redeems half of their common stock for a ten year loan with ten equal principal repayments.¹

Alternatively, in the third situation of Part C Table 15, the heir redeems half of their equity interest for an investor interest in a ten year loan (or a bond) of equal fair market value. The loan or bond pays

¹When the heir redeems half of their equity ownership for an investor interest in the corporation, the proportionate shares of equity ownership of the parents and heir increase due to the decrease in the number of shares outstanding. If initially there are 100 shares of common outstanding of which the parents own 80 and the heir then converts 10 of his shares into debt, this leaves 90 shares outstanding. The parents' proportionate interest has increased to 80/90 or 89 percent while the heir's equity interest has decreased from 20/100 to 10/90 or 11 percent.

periodic interest on the principal over its term. Similarly, the parents can redeem half of their common stock for either an investor interest in a ten year loan or a ten year bond.

The interest rates on the loan or bond are set at two levels, as described in Part D of Table 15.¹ The rates selected are 12 percent, which is assumed to be the market rate of interest, and 9 percent, which is below the market rate. With either interest rate, the loan or the bond is convertible into the same number of common shares. That is, the market value of the loan or bond is not adjusted for a change in the interest rate. Finally, in Part E of Table 15, the base situations are run with two income distribution plans. In the one salaries are paid to each of the family members but no dividends paid on the common stock. In the other, all of the net income of the farm corporation is distributed with dividends.

Results of the Iowa State University Business and Financial Model are illustrated in the following section.

Illustrative Results

The Iowa State University Business and Financial Planning Model generates a statement of financial position, a statement of changes in financial position, and individual cash flow statements for each of the

¹There are proposed regulations pending that would impose specific limitations on interest payments on debt instruments issued by a farm corporation. For more detail, see "Changes in Tax Laws" in the foreward of this study.

ten years in the planning horizon. The model recursively calculates Equations (5-1) through (5-33) for the farm firm and the family members. In addition, the model performs these calculations for four legal forms of business organization--the sole proprietorship, the partnership, the subchapter S corporation, and the subchapter C corporation.

Tables 16 and 17 present selected results from the detailed analysis generated by the model. The initial situation in each table is a class IV farm owned 80 percent by the parents and 20 percent by the heir. From Table 7, page 176, and from Table 10, page 181, a class IV farm has \$964,198 in assets and \$192,840 in liabilities in 1980. The firm's net equity in 1980 of \$771,358 is distributed \$617,086 (80%) to the parents and \$154,271 (20%) to the heir as shown in Table 16. Over the ten year planning horizon, the parents' equity grows to \$1,941,062, and the heir's equity grows to \$485,265 for a combined equity of \$2,426,327. In 1989, the combined assets of the firm are \$3,032,910 and combined liabilities are \$606,583. Over the ten year period, equity has grown at an average annual rate of 12.1 percent. Accumulated corporate income taxes for the firm are \$67,020 and represent, on average, 18.4 percent of corporate income. The parents paid accumulated personal income taxes of \$29,316, or 14.7 percent of income, while the heir paid \$14,658 in taxes. Total accumulated taxes for the firm and family are \$110,994 for an average 16.7 percent of combined income. On average, the parents' annual expenditures for living expenses are \$15,470, the heir's are \$7,735 for a family total of \$23,205.

Table 16. Illustrative results of the Iowa State University business and financial planning model: example one

		A class IV farm, 80% owned by the parents		
		Parents	Heir	Combined
Equities				
Total Assets, 1980		\$ 771,358	\$192,839	\$ 964,198
Total Liabilities, 1980		154,272	38,568	192,840
Net Equity, 1980		617,086	154,271	771,358
Total Assets, 1989		2,426,328	606,582	3,032,910
Total Liabilities, 1989		485,266	121,316	606,583
Net Equity, 1989		1,941,062	485,265	2,426,327
Average Annual Growth Rates		12.1%	12.3%	12.1%
Income Taxes	Corporate			
	Accumulated	\$67,020	\$ 29,316	\$ 14,658
	Average Percent of Income	18.4%	14.7%	14.7%
Average Annual Living Expenses		\$ 15,470	\$ 7,735	\$ 23,205
Comparative Statement of Changes in Financial Position, 1985				
Net Income		\$ 26,392	\$ 6,598	\$ 32,990
Income Taxes (Corporate)		4,678	1,170	5,848
Investment in Business Assets		41,220	10,305	51,525
Inflationary Gain		102,196	25,549	127,745
Individual Cash Flows, 1985				
Salary, Dividends, Directors' Fees		\$ 6,384	\$ 3,192	\$ 9,576
Off-Farm Income		12,842	6,421	19,263
Personal Income Taxes		2,614	1,307	3,921
Living Expenses		15,580	7,790	23,370
Net Cash Flow		1,030	515	1,545
Investment in Farm		1,015	500	1,515
Investment in Personal Assets		15	15	30

Table 17. Illustrative results of the Iowa State University business and financial planning model: example two

		A class IV farm, 80% owned by the parents--the heir redeems half his common for a 12% bond		
		Parents	Heir	Combined
Equities				
Total Assets, 1980		\$ 858,136	\$106,062	\$ 964,198
Total Liabilities, 1980		240,278	29,697	269,975
Net Equity, 1980		617,088	77,135	694,223
Investor Interest, 1980		0	77,135	
Total Assets, 1989		2,495,862	480,598	2,976,460
Total Liabilities, 1989		499,172	96,120	595,292
Net Equity, 1989		1,996,690	384,478	2,381,168
Investor Interest, 1989		0	0	
Average Annual Growth Rates		12.5%	17.4%	13.1%
		Corporate		
Income Taxes				
Accumulated	\$47,284	\$ 40,196	\$ 38,026	\$ 125,506
Average Percent of Income	17.7%	20.3%	19.9%	19.1%
Average Annual Living Expenses		\$ 15,468	\$ 7,919	\$ 23,385
Comparative Statement of Changes in Financial Position, 1985				
Net Income		\$ 19,546	\$ 3,764	\$ 23,310
Income Taxes (Corporate)		3,322	640	3,962
Purchases of Business Assets		41,520	7,995	49,515
Inflationary Gain		106,755	20,557	127,312
Individual Cash Flows, 1985				
Salary, Dividends, Directors' Fees		\$ 6,384	\$ 3,192	\$ 9,576
Off-Farm Income		12,748	6,374	19,122
Interest Income		0	9,252	9,252
Personal Income Taxes		3,428	3,372	6,800
Living Expenses		15,578	7,974	23,552
Net Cash Flow		124	7,471	7,595
Investment in Farm		122	7,108	7,230
Investment in Personal Assets		2	363	365

In the lower half of Table 16, key results from the comparative statement of changes in financial position are presented for 1985.¹ Net income from operations is \$32,990 with corporate income taxes of \$5,848. Purchases of new business assets are \$51,525. Revaluation of the firm's assets to fair market value results in a \$127,745 inflationary gain.² Since the parents are assumed to own 80 percent of the firm, 80 percent of the changes in financial position are attributable to them. The individual cash flows for 1985 are shown in the bottom section of Table 16. The parents receive \$6,384 in salaries, dividends, and director's fees from the firm while the heir receives \$3,192. Off-farm income for the parents is \$12,842 and for the heir, \$6,421.³ The parents paid personal income taxes of \$2,614 and living expenses of \$15,580. The heir paid personal income taxes of \$1,307 and living expenses of \$7,790. Of the parents' net cash flow of \$1,030, \$1,015 is reinvested in the firm. The heir reinvests \$500 of his (her) net cash flow of \$515.

¹Since the model does not calculate average annual figures for the statement of changes in financial position or the cash flow statements, the middle year of the planning horizon (1985) is used as a proxy for the average annual values.

²Other line items from the statement of changes in financial position not presented here are depreciation, proceeds on sale of assets, loss or gain on sale of assets, new borrowings, additions to contributed capital, debt retirements, and withdrawals.

³Off-farm income includes all income not directly attributable to the farm entity. In addition to off-farm wages and salaries, this category includes interest and dividends received on savings and also off-farm investments and net proceeds from performing custom hire work.

Table 17 initially begins with the same situation as did Table 16 except now the heir redeems half his (her) common stock for a \$77,135 ten-year bond paying 12 percent interest. The firm's liabilities increase from \$192,840 to \$269,975 and equity decreases from \$771,358 to \$694,223. The heir's equity is cut in half, thereby reducing the total number of shares outstanding and altering the proportionate ownership of the parents. Before the heir redeemed his (her) stock, the parents' equity was worth \$617,086 or 80 percent of \$771,358. After the redemption, the parents' 80 percent becomes 89 percent of \$694,223. In addition, the heir now has a \$77,135 investor interest in the firm. Ten years later, the parents' equity is larger than before (\$1,996,690 versus \$1,941,062), while the heir's is smaller than before (\$384,478 versus \$485,265).¹ Because of the tax deductibility of interest payments on the bond, corporate accumulated taxes are nearly \$20,000 lower in Table 17 than they were in Table 16. Correspondingly, the heir's accumulated personal income taxes are higher in Table 17 because the interest is taxed as personal income. The combined accumulated taxes for the firm and family members increase from \$110,994 to \$125,506 with the conversion to a ten year bond. In 1985, net income from operations is reduced to \$23,310 by the interest payment on the bond. Corporate taxes in 1985 are \$3,962. The major difference in the cash flow statements from Table 16 to Table 17 is the \$9,252 in bond interest received by the heir. The

¹The heir's average annual growth rate is larger than before (17.4% in Table 17 versus 12.3% in Table 16) but this is because the heir starts with a smaller equity in Table 17.

change in cash flow increases the heir's personal income taxes from \$1,307 to \$3,372 for 1985.

By comparing Table 16 with Table 17, it is obvious that the change in the parents' net equity increases when the heir converts half his (her) stock to a bond. This increase in wealth for the parents comes at the expense of the heir. An heir wishing to maximize his (her) utility would be better served by not redeeming common stock for a bond paying the market rate of interest because the interest earned on the bond is taxed currently whereas the appreciation on common stock is accrued as capital gain and not taxed until the stock is sold.

The results presented in Table 16 and 17 presume that expectations are realized--that is, there is no uncertainty. It is now appropriate to expand the analysis by letting the actual outcomes realized vary from the outcomes expected to occur.

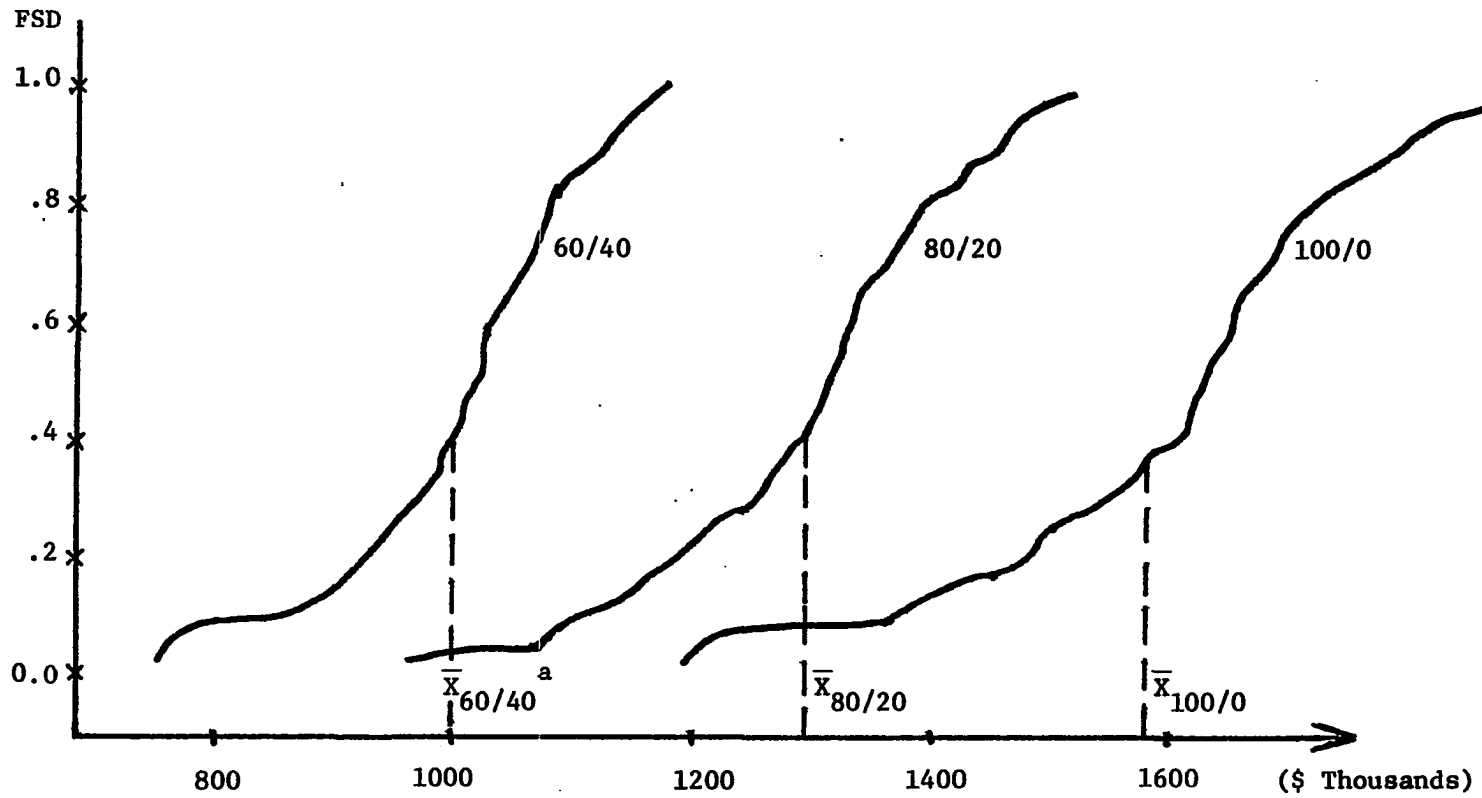
Recall that the objective function of the parents (and the heir is to maximize their net wealth at the end of the planning horizon. Because of the difficulty in constructing a family objective function, interpersonal comparisons can not be accomplished. Rather, outcomes that produce conflict between the parents and the heir can be identified but not resolved. Outcomes which don't produce conflict (but increase both the parents' and the heir's net terminal wealth) facilitate the intergenerational transfer process.

Risk is incorporated into the analysis by measuring the change in net wealth that results from a selected financing arrangement as a cumulative density function. Each cumulative density function is generated

by recursively performing the detailed analysis equivalent to that presented in Tables 16 and 17 with a Monte Carlo simulation driven by a random number generator. Each point on a graph represents one simulation for one time pattern of stochastic error terms. Each graph measures the change in wealth over the ten-year planning horizon for the parents or the heir for a selected financing arrangement under many randomly determined time patterns of stochastic error terms. After a sufficient number of analyses are performed, the results are ranked in ascending order and cumulatively summed to derive the cumulative density functions discussed in the remainder of this chapter.

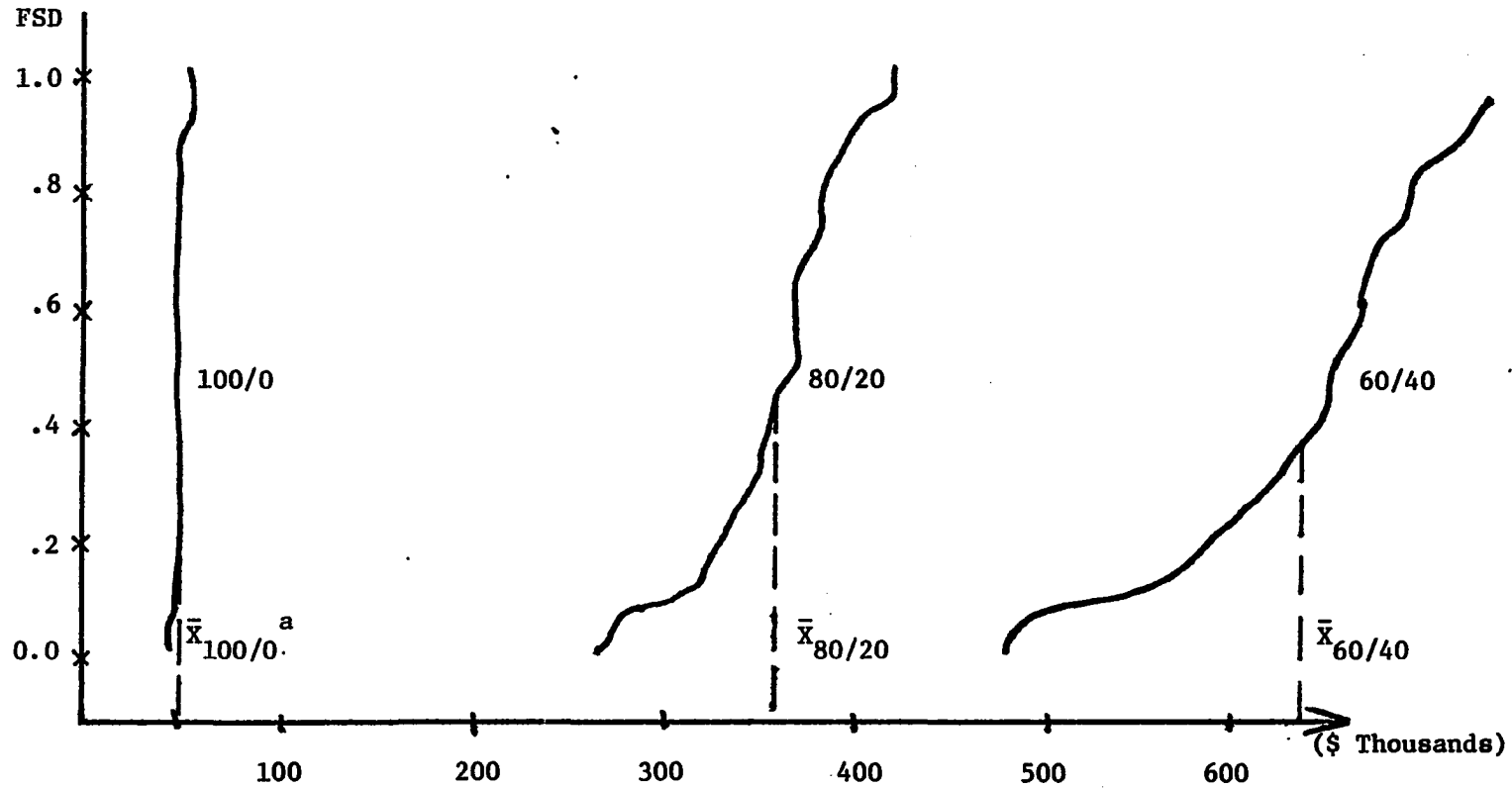
Ownership Mix

Figures 30 through 37 present the cumulative density functions for both the parents and the heir for each of the representative farms for each of the three ownership mixes. The three initial ownership mixes for a class II farm are graphed in Figure 30 for the parents and Figure 31 for the heir. Curve 100/0 on Figures 30 and 31 represent the parents' ownership of 100 percent of the common stock and 0 percent for the heir. Curve 80/20 on Figures 30 and 31 represent 80 percent ownership by the parents and 20 percent ownership by the heir, and curve 60/40 represents the 60/40 initial mix. Similarly, a class III farm is depicted in Figures 32 and 33, a class IV farm in Figures 34 and 35, and a class V farm in Figures 36 and 37.



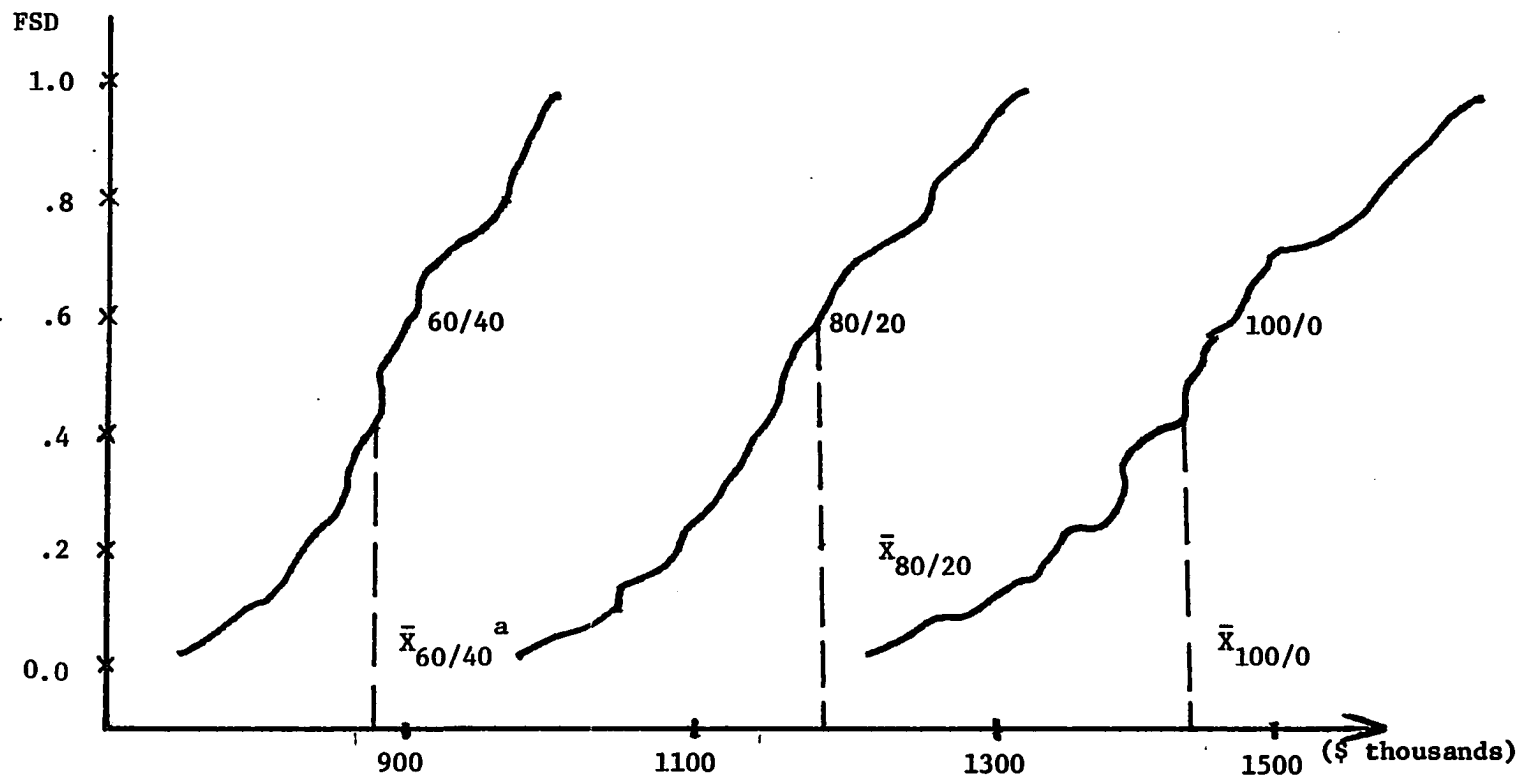
^a Mean values.

Figure 30. A comparison of initial ownership mixes for the parents of a Class II farm with common stock only



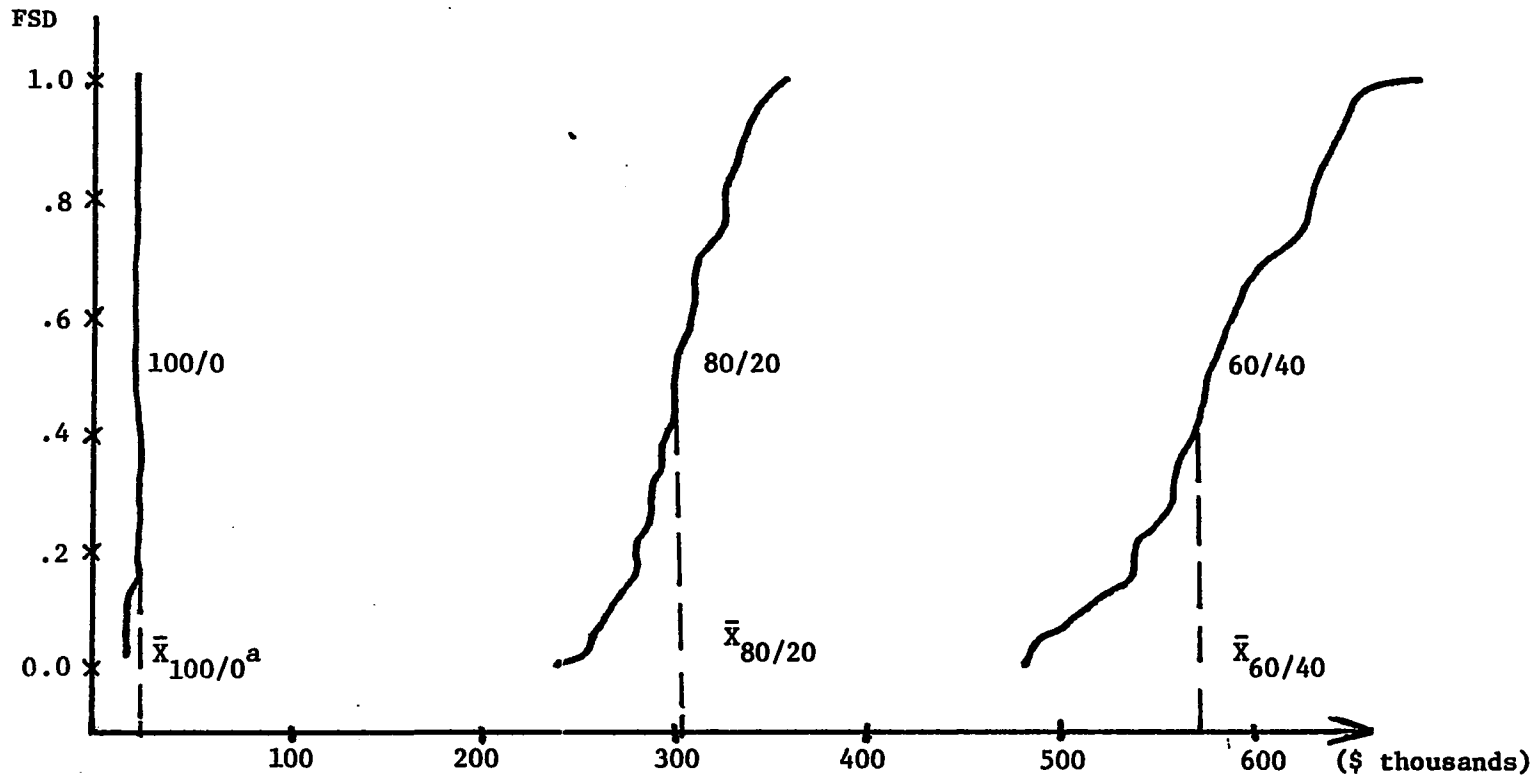
^a Mean values.

Figure 31. A comparison of initial ownership mixes for the heir of a Class II farm with common stock only



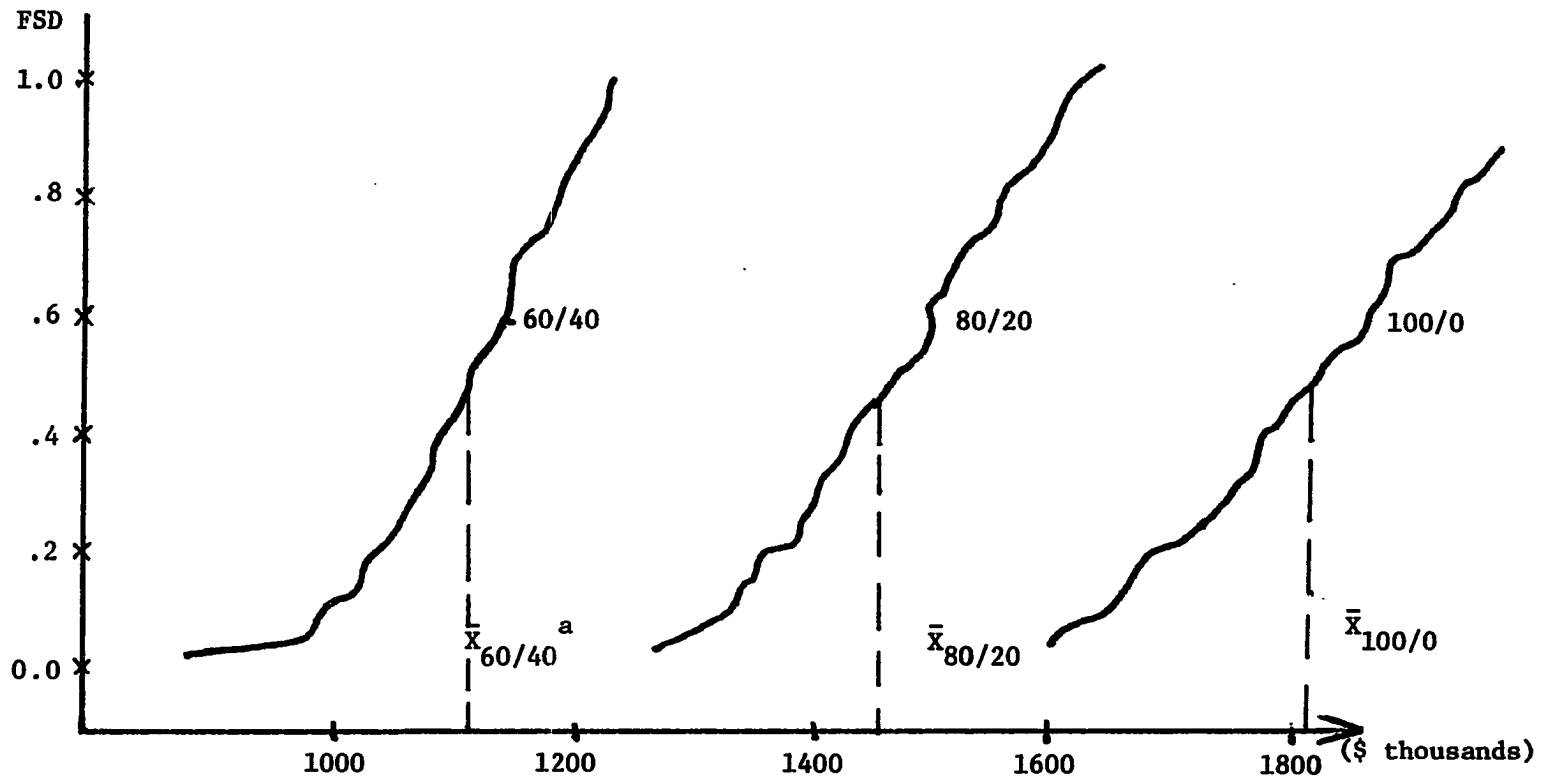
^a Mean values.

Figure 32. A comparison of initial ownership mixes for the parents of a Class III farm with common stock only



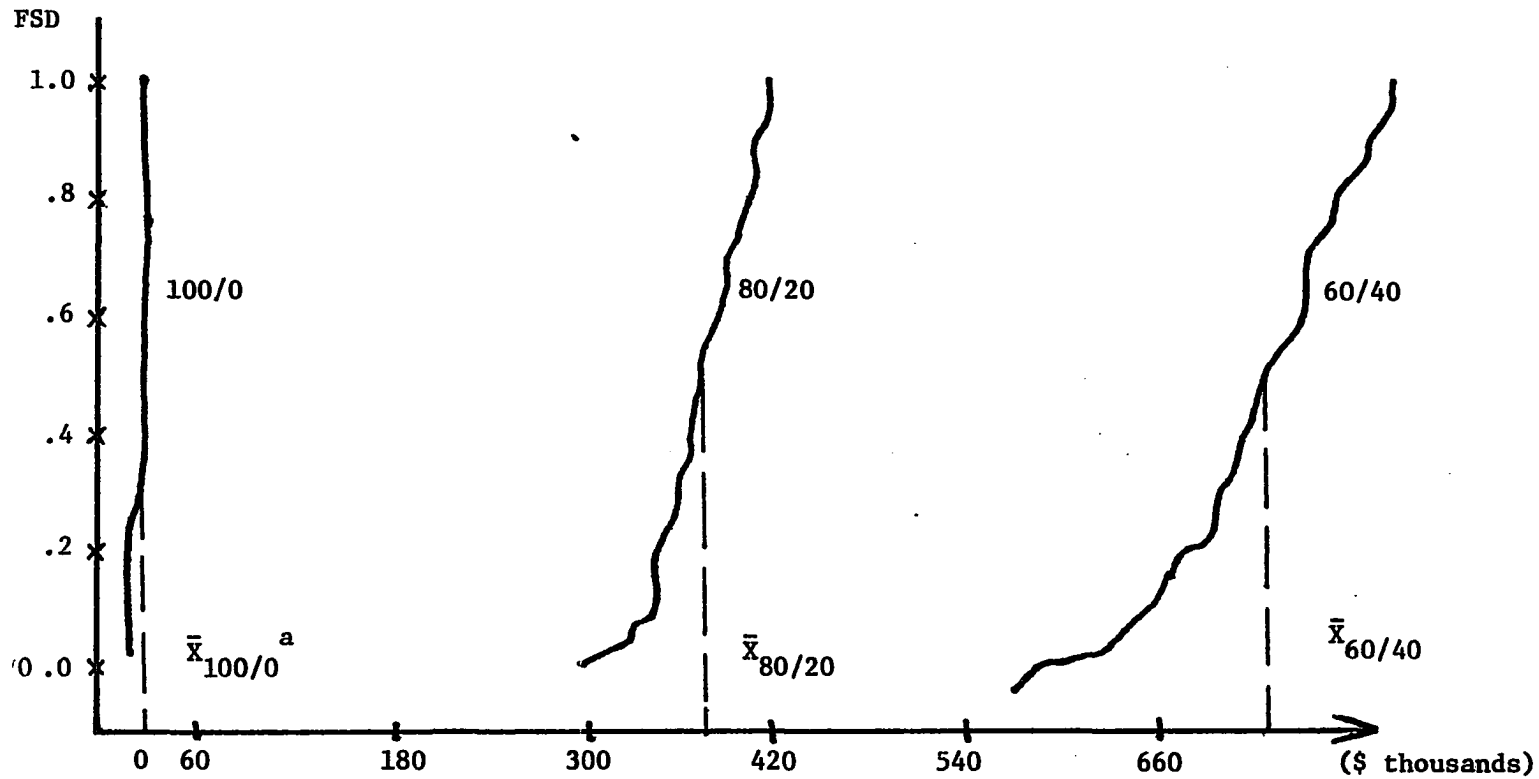
^a Mean values.

Figure 33. A comparison of initial ownership mixes for the heir of a Class III farm with common stock only



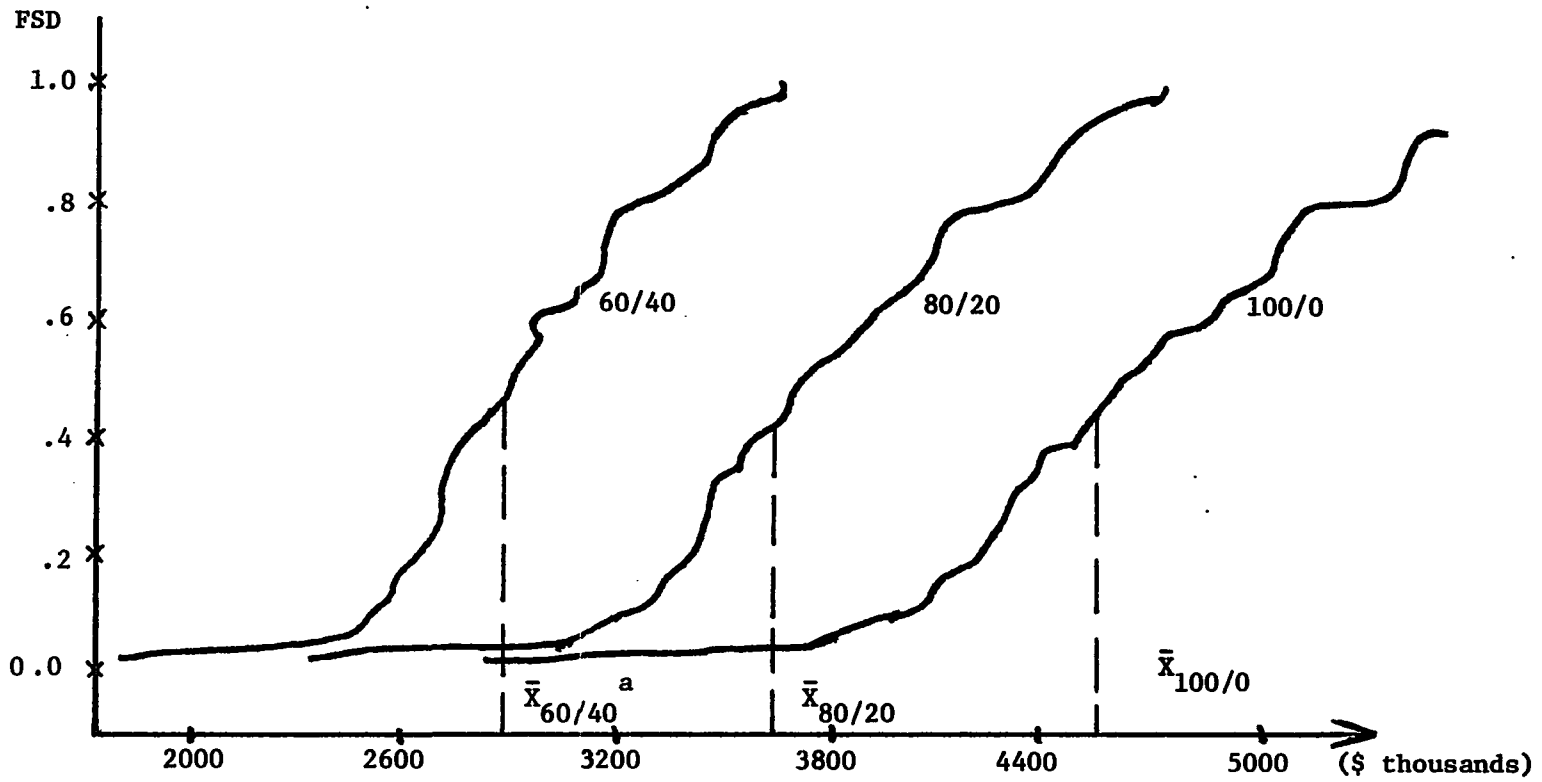
^a Mean values.

Figure 34. A comparison of initial ownership mixes for the parents of a Class IV farm with common stock only



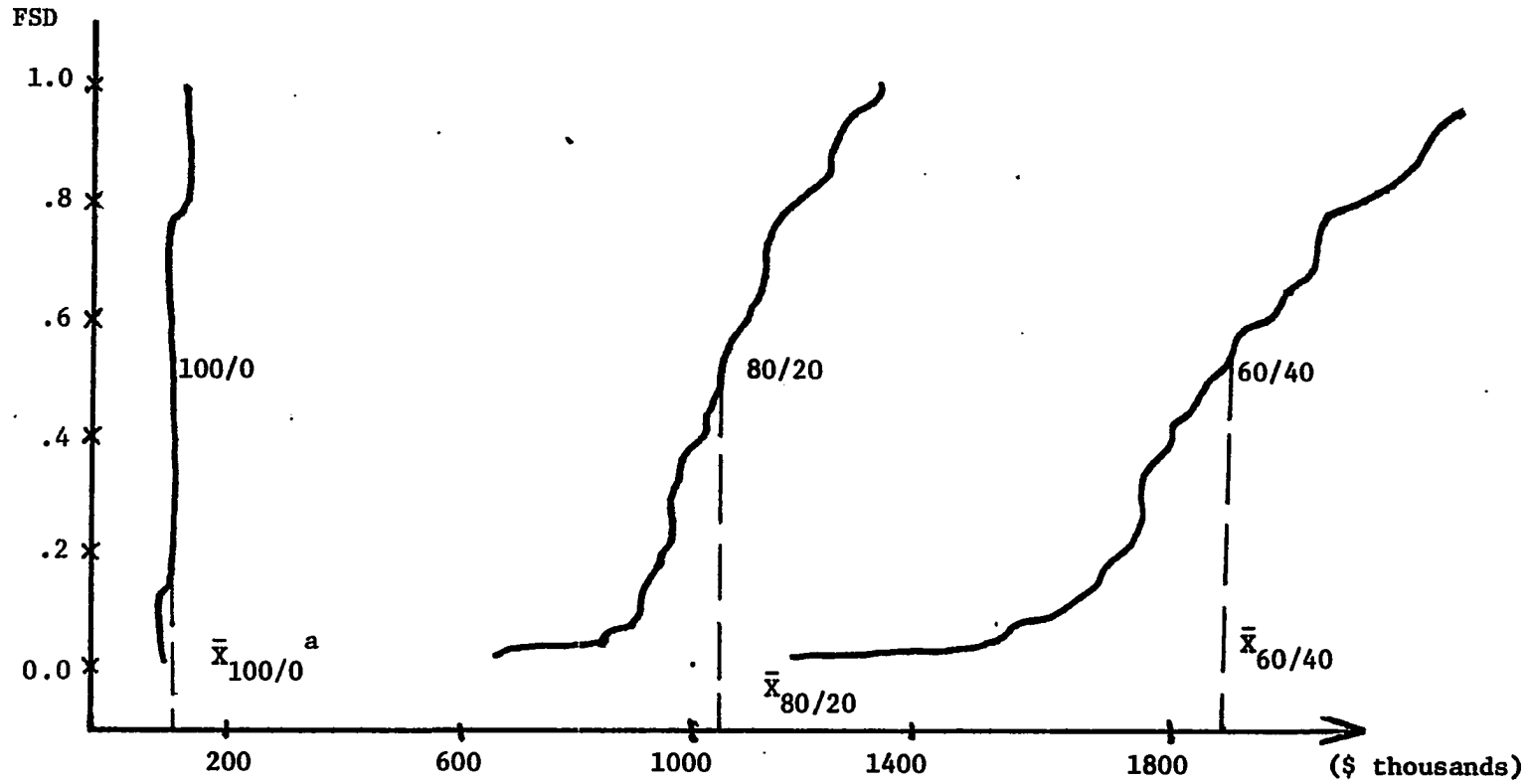
^a Mean values.

Figure 35. A comparison of initial ownership mixes for the heir of a Class IV farm with common stock only



^a Mean values.

Figure 36. A comparison of initial ownership mixes for the parents of a Class V farm with common stock only



^a Mean values.

Figure 37. A comparison of initial ownership mixes for the heir of a Class V farm with common stock only

As can be seen from Figures 30, 32, 34 and 36, the parents unambiguously prefer the 100 percent ownership to 80 percent to 60 percent ownership under first degree stochastic dominance. This is because first degree stochastic dominance (FSD) requires the preference of more wealth to less as an axiom of rational behavior. Similarly the heir, in Figure 31, 33, 35 and 37, unambiguously prefers the 60/40 to the 80/20 to the 100/0 distribution of ownership under FSD or just the opposite ordering of the parents.

There are two reasons for presenting these figures. First, the curves in Figures 30 through 37 serve as a basis for comparison for later situations and are presented here for completeness. Second, Figures 30 through 37 present the conflict of interests between parents and heir concerning intergenerational transfer. If the parents are not concerned with facilitating the intergenerational transfer, that is, the parents' objective of maximizing expected utility is not constrained by a desire to transfer the farm to the heir, then the parents will always prefer owning the entire farm and will not gift any of their interest in the farm. Conversely, the heir wishes to own as much of the farm as possible since this maximizes his (her) utility. Therefore, potential conflict exists between the parents and the heir.

Salaries Versus Dividends

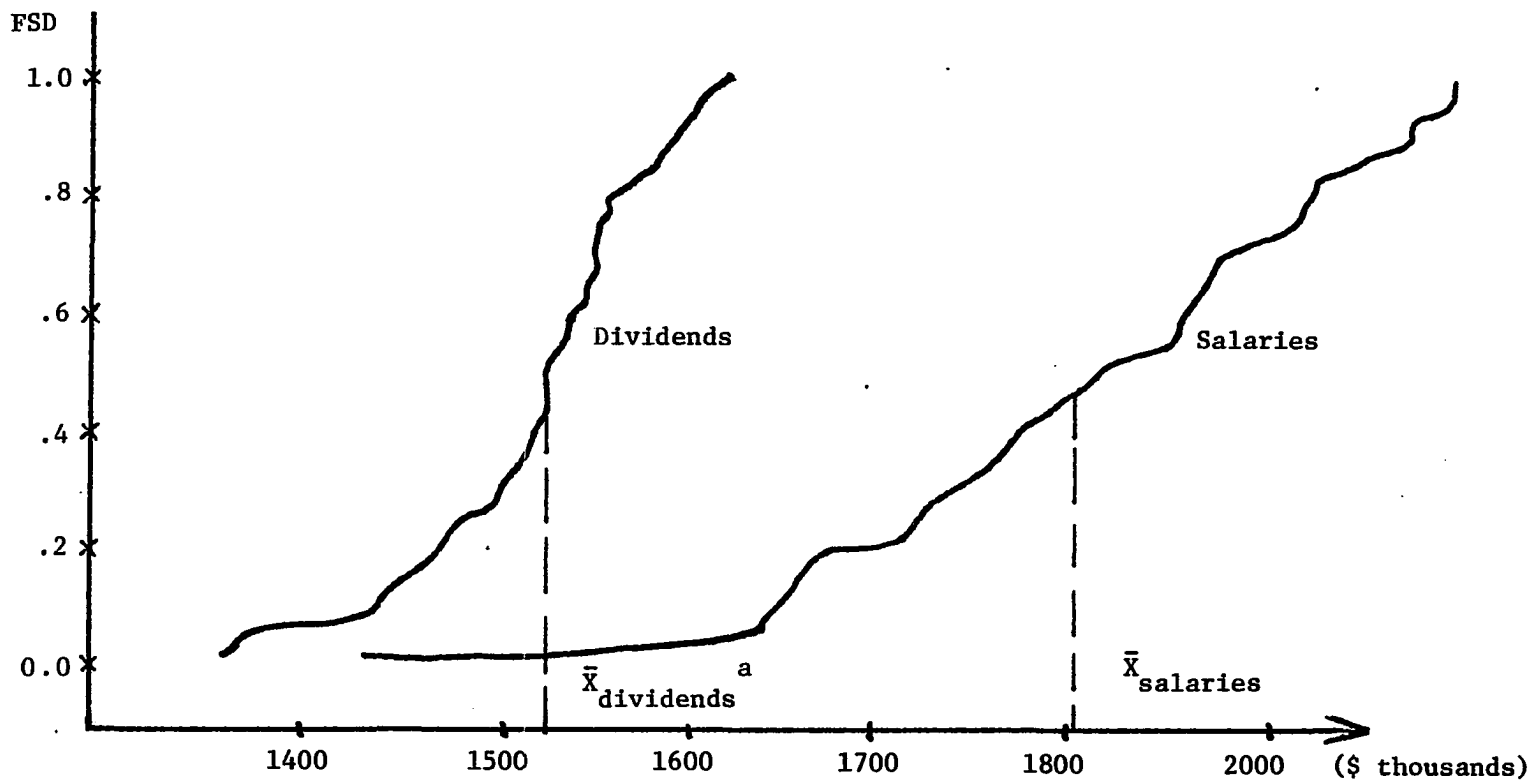
Common methods of sharing income between the parents and the heir are through the use of salaries, directors' fees, dividends on common stock, and interest payments on debt. Salaries and directors' fees are

compensation for the contribution of personal labor and managerial expertise, while dividends are compensation for the contribution of physical and money capital to the firm. Interest payments are compensation for nonequity investments in the firm.

Salaries which meet the Internal Revenue Service criterion for reasonableness are tax deductible to the corporation as a valid business expense. The salary received by an individual must be reported as taxable income on the individual's federal income tax return. Dividends are distributed from the after-tax earnings of the corporation. However, the first \$400 (on a joint return) of taxable dividends and interest received is tax deductible to the individual. Interest payments are deductible by the corporation but the excess over \$400 received is taxable to the individual.¹ Therefore, income of the corporation is taxed only once if it is distributed as either salaries or interest (except for the first \$400 of interest, which is not taxed at all), but it is taxed twice (except for the first \$400) if distributed as dividends.

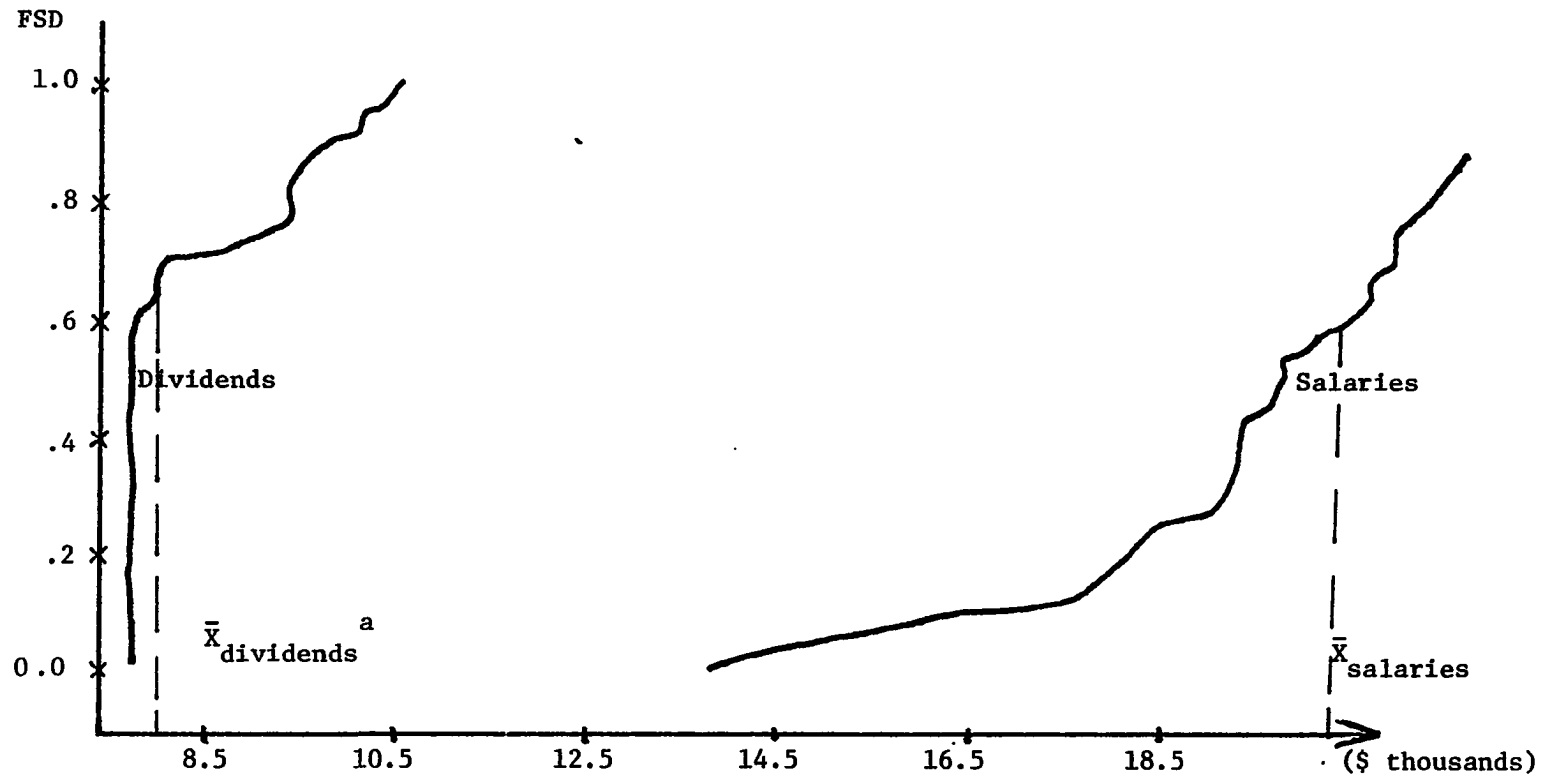
When salaries or dividends can accomplish the family's desired income sharing plan equally well, salaries would be preferred since they produce a smaller combined tax liability. As shown in Figures 38 through 41, this is exactly what happens. In Figure 38, the parents in a Class

¹For tax years beginning after 1981, there is no exclusion for interest received. The exclusion on a joint return is \$200 for qualifying dividends regardless of which spouse has legal title to the stock (25b, p. 246).



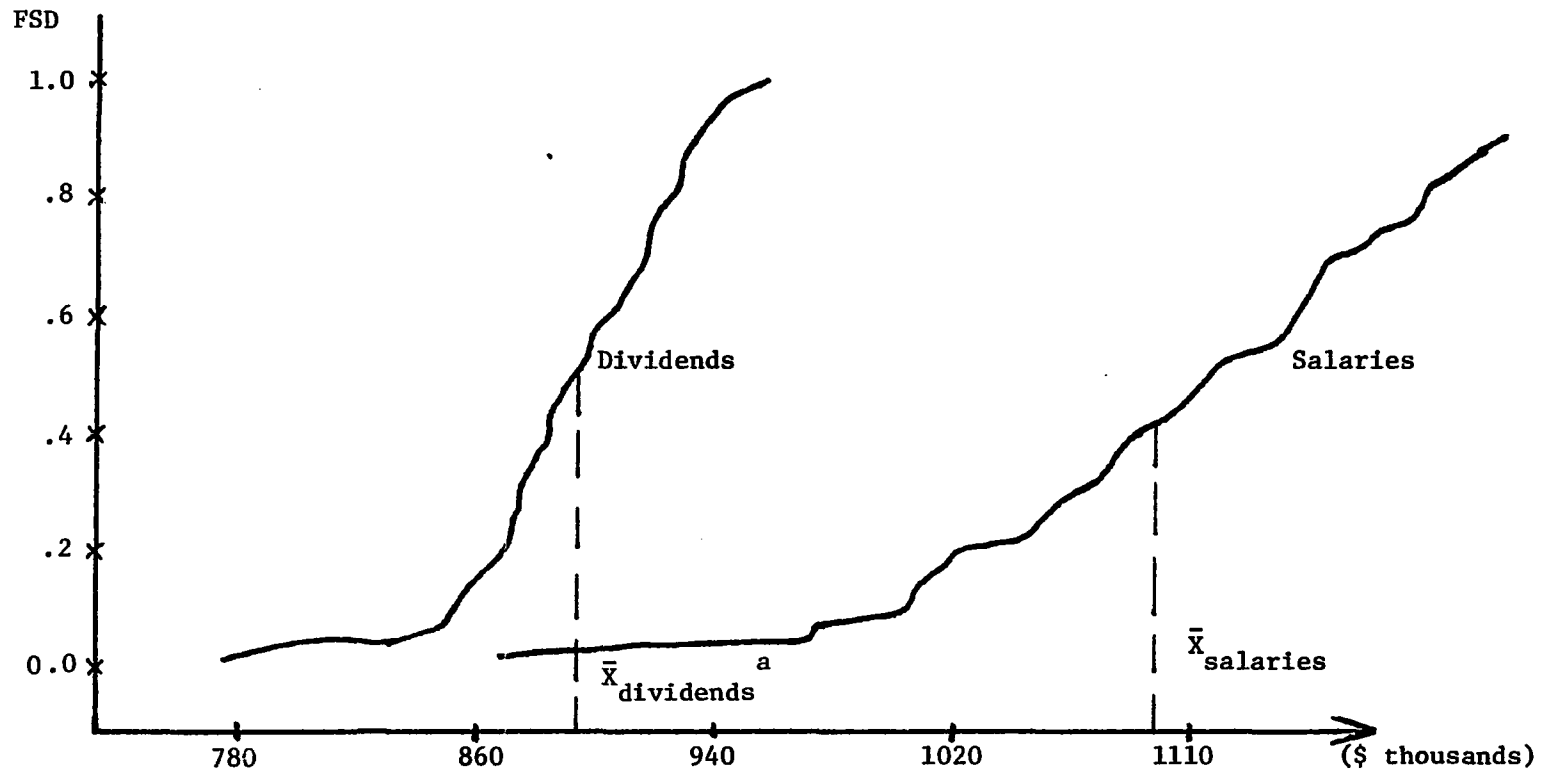
^a Mean values.

Figure 38. A comparison of salaries and dividends for the parents of a Class IV farm with 100 percent initial ownership



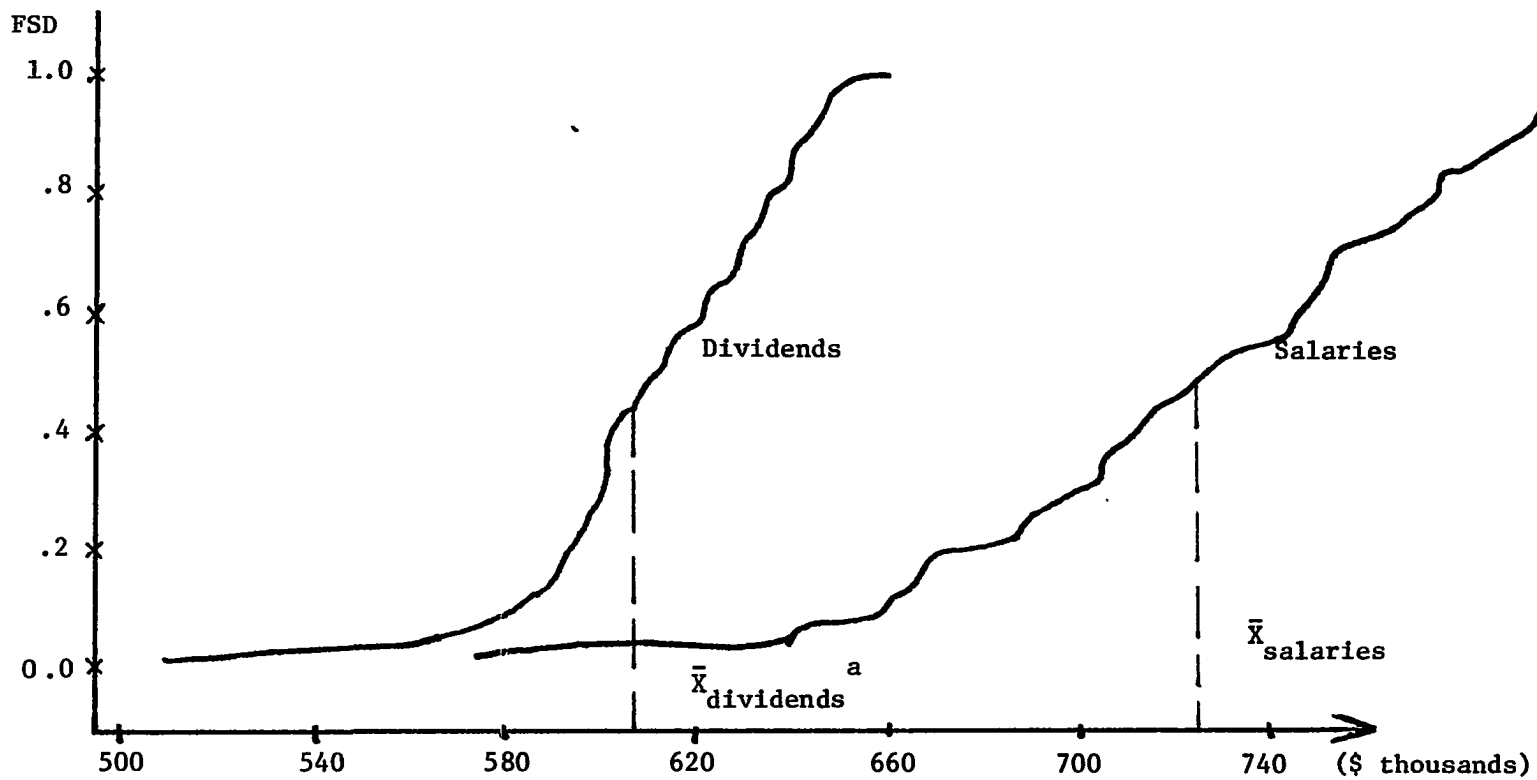
^a Mean values.

Figure 39. A comparison of salaries with dividends for the heir of a Class IV farm who initially owns 0 percent.



^a Mean values.

Figure 40. A comparison of salaries with dividends for the parents of a Class IV farm who initially own 60 percent of it



^a Mean values.

Figure 41. A comparison of salaries with dividends for the heir of a Class IV farm who initially owns 40 percent

IV farm with 100 percent ownership prefer salaries to dividends by FSD. In Figure 39, the heir also prefers salaries to dividends under FSD in a Class IV farm when the parents own all of the farm. In Figure 38, changing the income sharing plan from stock dividends to salaries increases the mean change in the parents' terminal equity from \$1,518,100 to \$1,807,600, or by 19 percent.¹ This increase in equity is attributable to the double taxation of dividends. Figure 40 represents the change in the parents' terminal wealth distribution under FSD for a Class IV farm with 60 percent ownership; in all cases the parents prefer salaries to dividends under FSD. Figure 41 summarizes the wealth distribution of the heir under FSD; as with the parents, the heir prefers salaries to dividends in every case.²

In the comparison of salaries with dividends, there is no conflict of interest between the parents and the heir. Both unambiguously prefer salaries to dividends under FSD. But this is not to say dividends are never important. To avoid double taxation, salaries must meet the test

¹However, the variability of the distribution also increases with the switch from dividends to salaries--from a standard deviation of \$62,300 to \$131,100. This increase in variability occurs because salaries are a fixed expense--the same salary is paid regardless of the level of after-tax income. Dividends, on the other hand, vary proportionately with the level of after-tax income. If, in a particular year, there is no (or negative) after-tax income, no dividends are paid whereas salaries would still be paid. However, as can be seen in Figures 38 to 41, the increase in wealth from using salaries instead of dividends more than compensates the parents and the heir for the increase in variability.

²If in fact salaries fail the test of reasonableness, the distributions are treated as dividends for tax purposes.

of reasonableness. If the salary is not reasonable, the dividends would be preferable since at least the first \$400 of dividends is tax deductible at the personal level. Furthermore, dividends can be used to distribute income in relation to ownership instead of management or contributed labor. This feature becomes of increasing importance as the parents enter retirement, since they are inclined to reduce their participation in the management of the firm. In this situation, it becomes difficult to justify the reasonableness of salaries while dividends can provide the parents with retirement income based upon their investment in the firm. When annual dividends or the annual salaries are below \$400, the family members will prefer dividends if their personal marginal income tax rate is greater than the marginal tax rate of the corporation. Conversely, if the corporation's marginal tax rate is higher, then salaries (or director's fees) are preferred.

Common Stock Versus a Constant Principal Loan at Market Rate of Interest

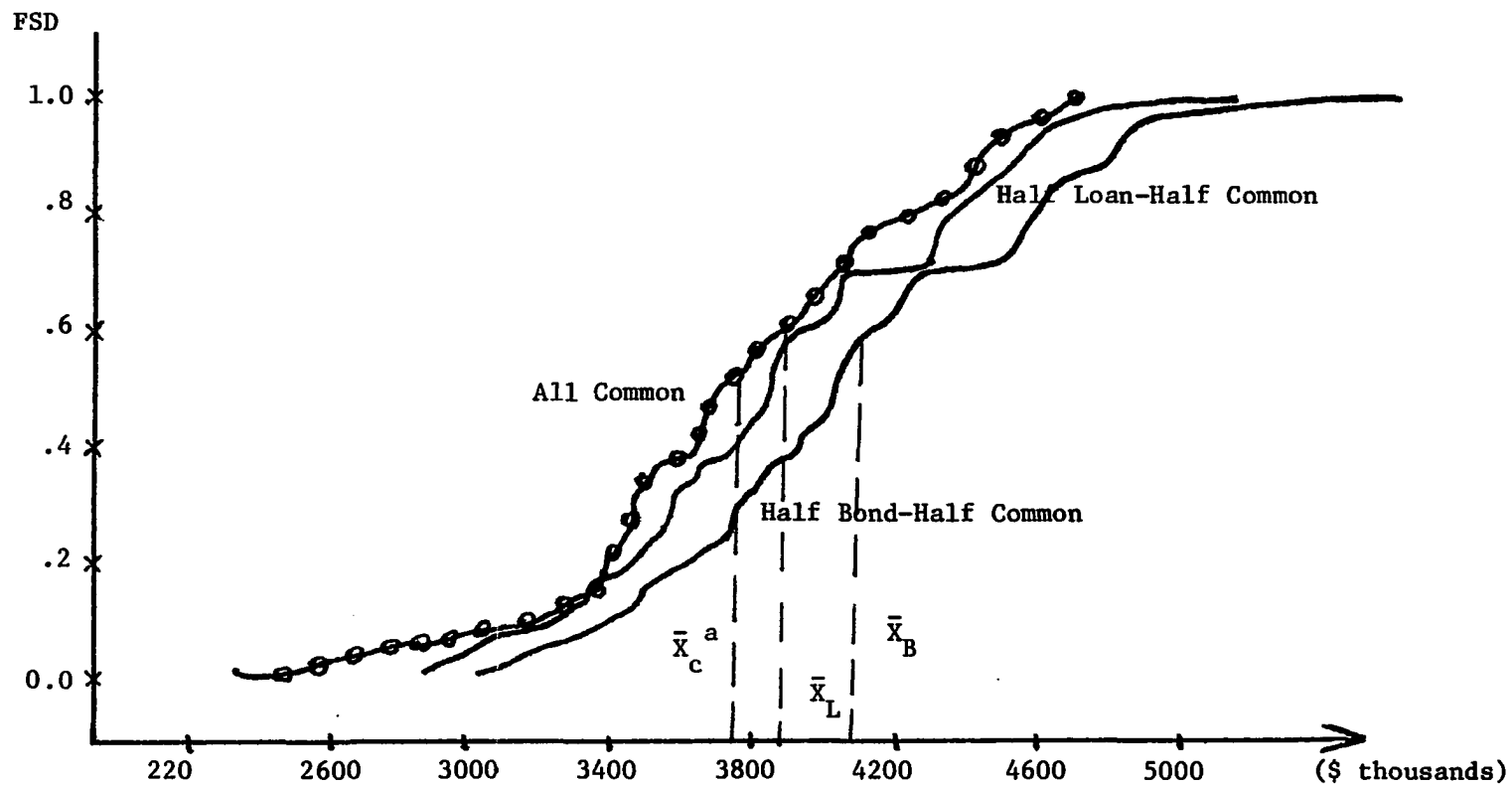
In this and the next three sections, either the parents or the heir redeem half their common stock for either a loan or a bond. Since the terms of the loan or bond result through direct negotiation without the involvement of a financial intermediary, the interest rate need not be determined by prevalent market conditions. Therefore, the stated rates of interest on the loan or bond are evaluated at 12 percent (assumed to represent the market rate) and 9 percent.

In each situation, the conversion of equity into debt has four ramifications on the growth in net terminal wealth of the parents and the heir:

1. A conversion of equity to debt by one party changes the proportionate equity ownerships of all parties (see footnote 1, page 193).
2. With the issuance of a new debt instrument, interest payments reduce after-tax corporate cash flow. As a result, the firm has less after-tax earnings available for distribution.
3. A conversion to a loan requires periodic principal payments to amortize the loan value to zero over the planning horizon. A conversion to a bond requires a single balloon payment of principal at maturity. As a result, the timing for reinvesting in the firm is altered.
4. The individual who holds the investor interest in the bond or loan enjoys a larger cash flow than he would had he kept his interest entirely in equity. This is due to the interest and principal payments he periodically receives.

Figures 42 through 51 summarize the changes in the net terminal wealth of the parents and the heir when the parents' initial ownership, or the heir's initial ownership, or both, is converted from an equity interest of common stock to a combined owner-investor interest. This is accomplished by redeeming half the initial common stock for a ten year, constant principal loan at the market rate of interest (assumed to be 12 percent).

Figure 42 summarizes the results for the parents who initially own 80 percent of a class V farm. The All Common curve in Figure 42 represents the situation where the heir's 20 percent interest is held entirely in common stock while the Half Common-Half Loan curve in Figure 42 represents the same circumstances except the heir redeems half his



^a Mean values: \bar{X}_c is the mean of All Common, \bar{X}_L is the mean of Half Loan-Half Common, and \bar{X}_B is the mean of Half Bond-Half Common.

Figure 42. A comparison of a loan with a bond, both at 12 percent, for the parents of a Class V farm who initially own 80 percent (all in common stock) and an heir who initially owns 20 percent (half in common stock and half in either a bond or a loan)

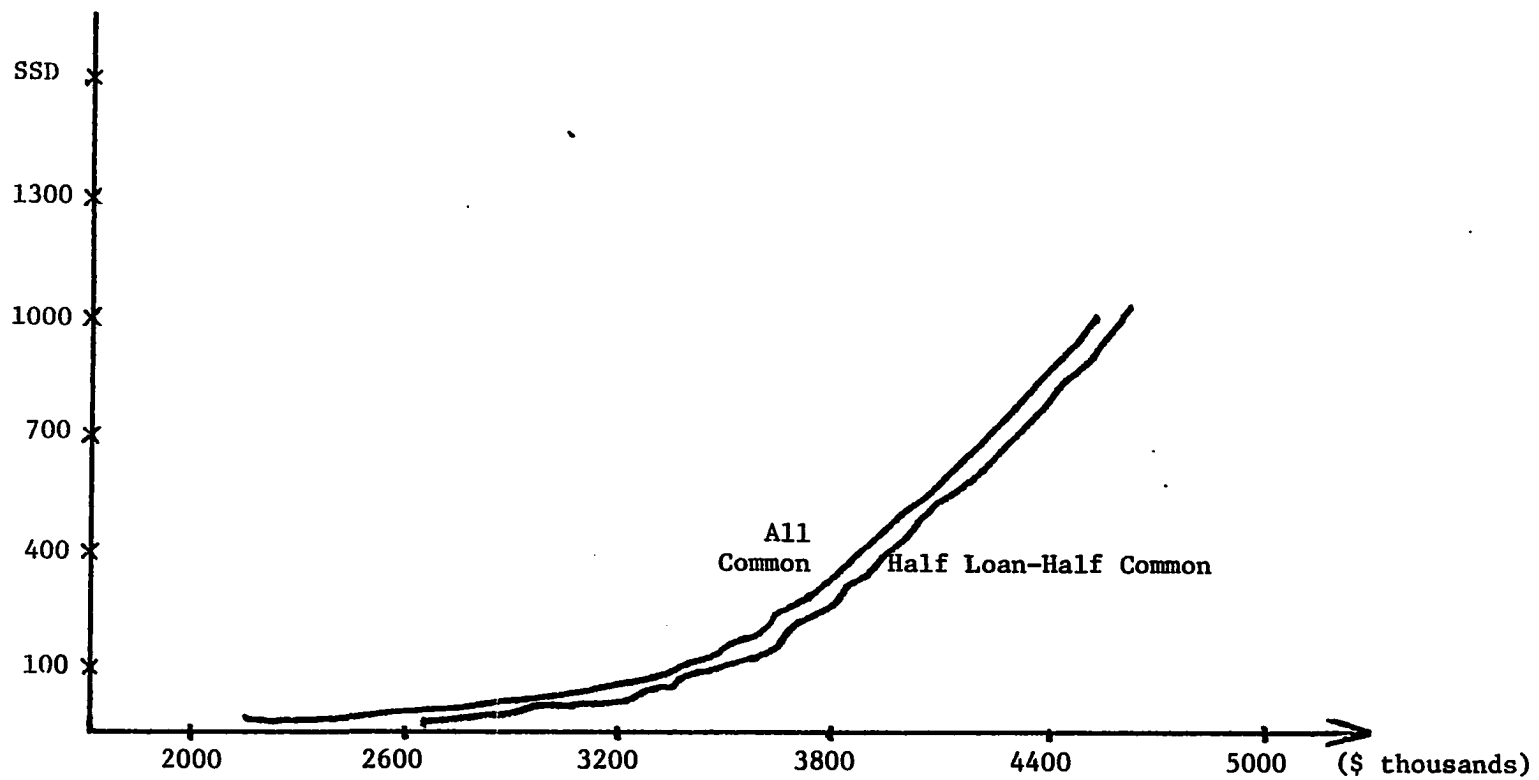
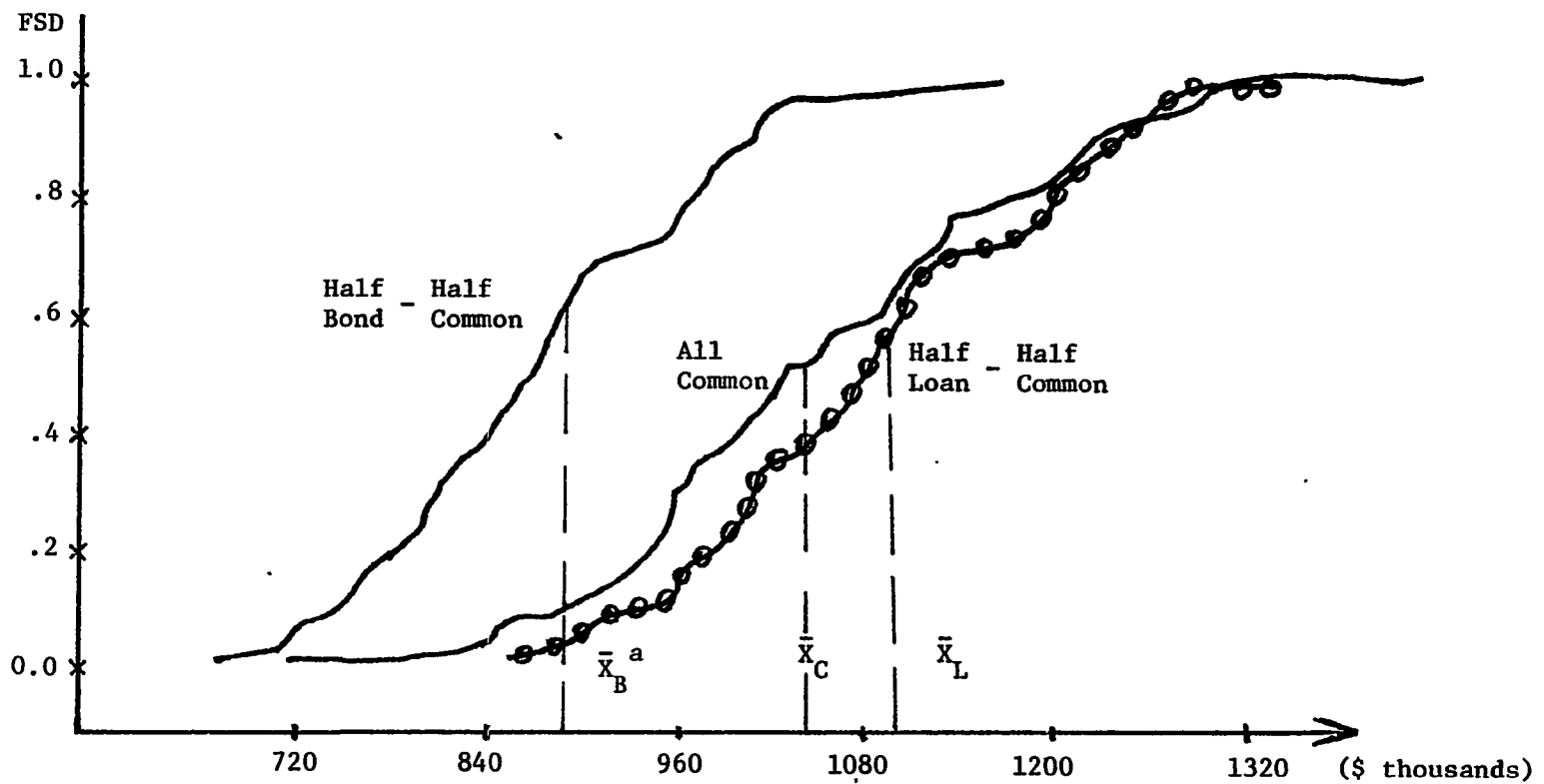


Figure 43. Second degree stochastic dominance of the situation shown Figure 42



^a Mean values: \bar{X}_C is the mean of All Common, \bar{X}_L is the mean of Half Loan-Half Common, and \bar{X}_B is the mean of Half Bond-Half Common.

Figure 44. A comparison of common stock with a loan with a bond, both at 12 percent, for the heir of a Class V farm who initially owns 20 percent, half in common stock and half in either common, a loan or a bond

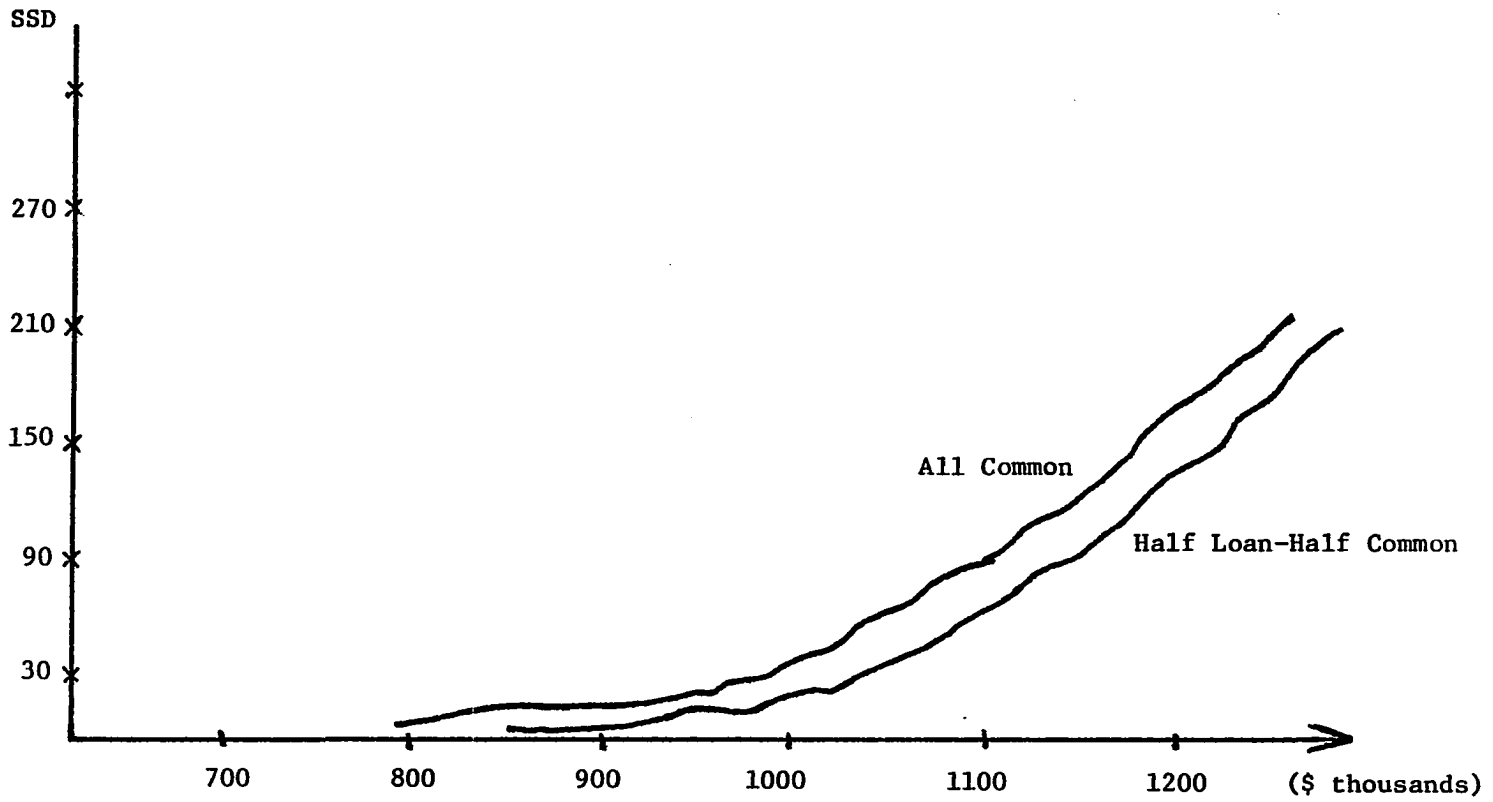
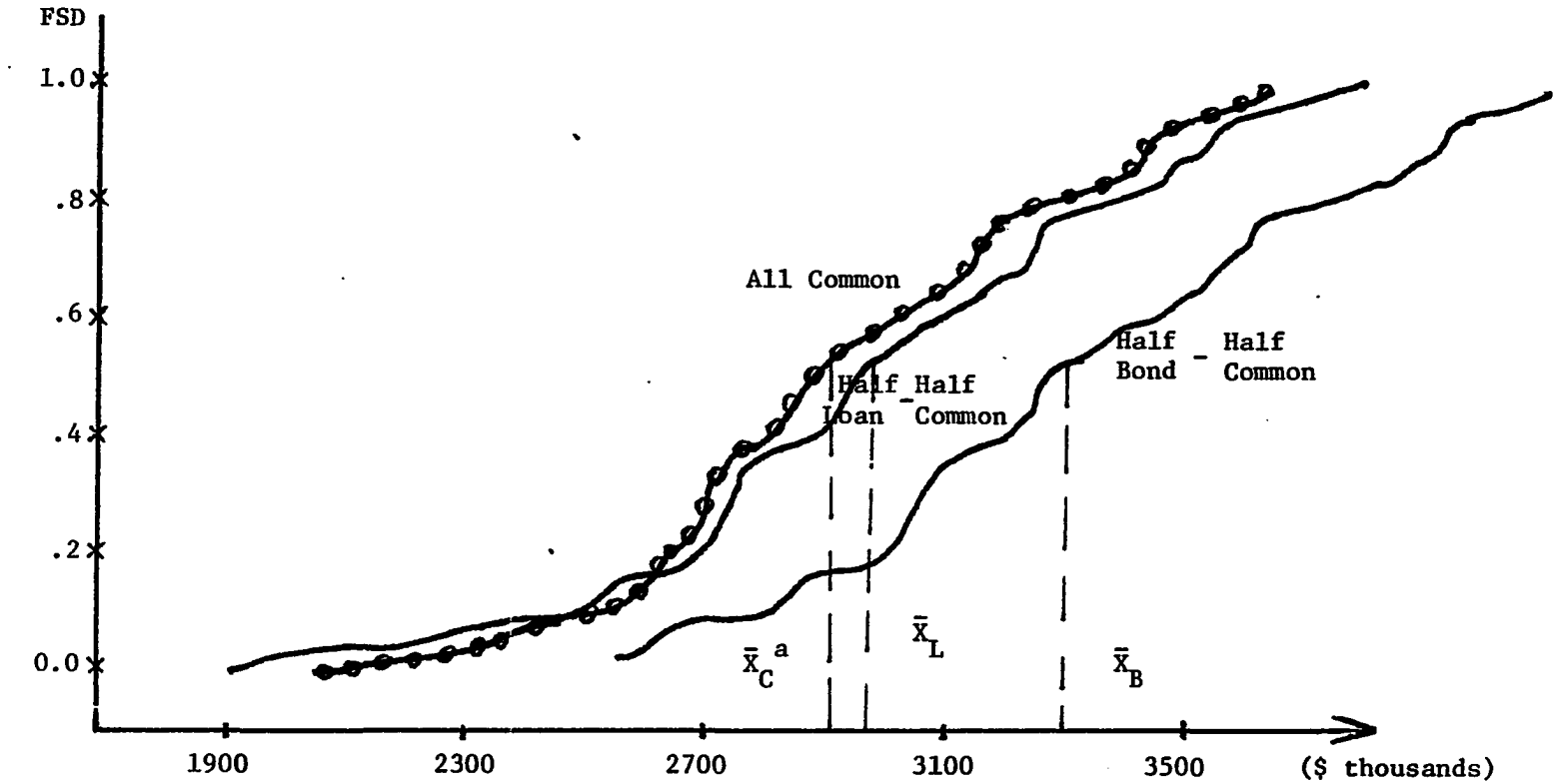


Figure 45. Second degree stochastic dominance of the situation in Figure 44



^a Mean values: \bar{X}_C is the mean of All Common, \bar{X}_L is the mean of Half Loan-Half Common, and \bar{X}_B is the mean of Half Bond-Half Common.

Figure 46. A comparison of common stock with a 12 percent loan with a 12 percent bond for the parents of a Class V farm who initially own 60 percent (all in common stock) and an heir who initially owns 40 percent (half in common and the other half in either common, a loan or a bond)

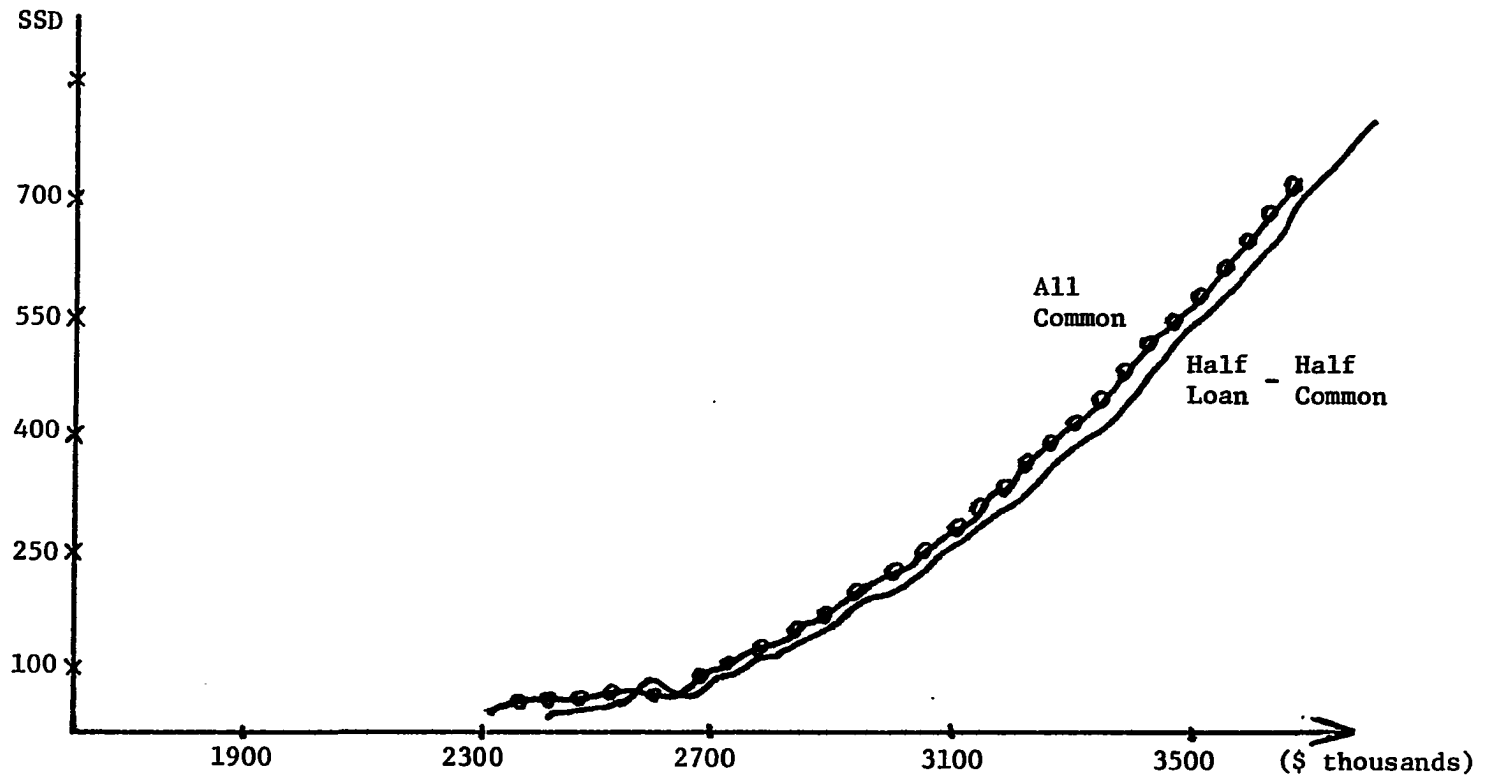


Figure 47. Second degree stochastic dominance of the situation shown in Figure 46

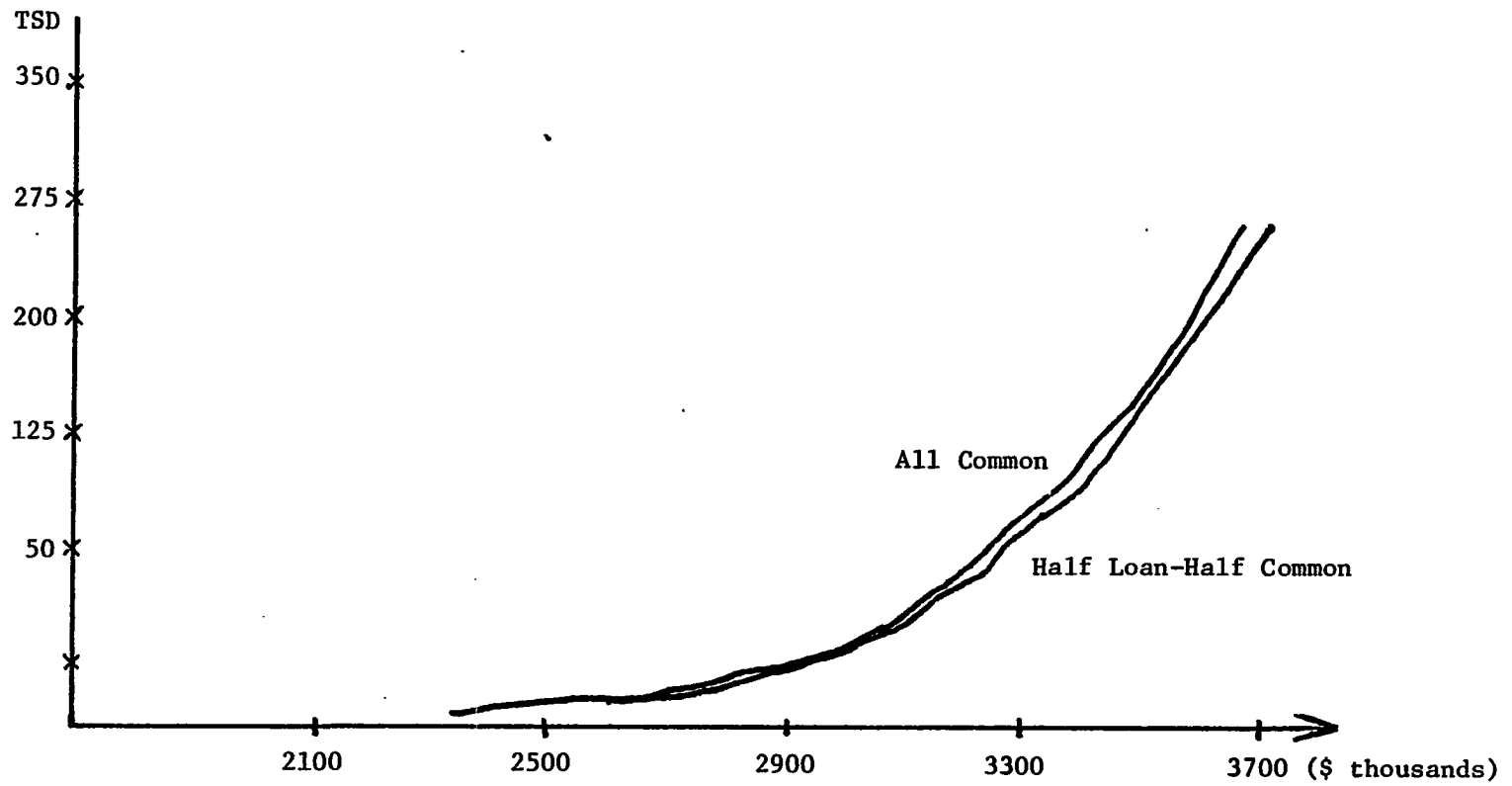
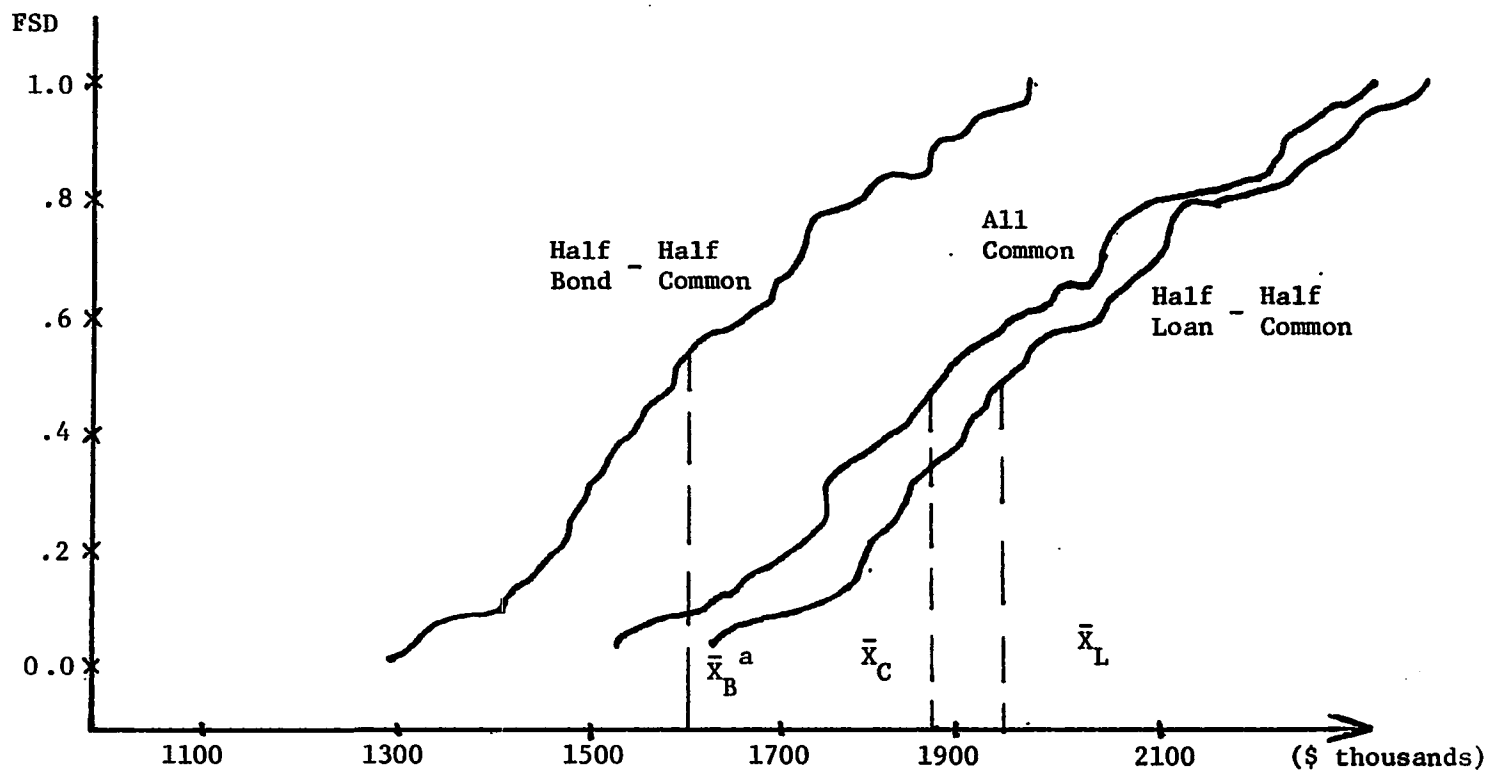


Figure 48. Third degree stochastic dominance of the situation shown in Figure 46



^a Mean values: \bar{X}_C is the mean of All Common, \bar{X}_L is the mean of Half Loan-Half Common, and \bar{X}_B is the mean of Half Bond-Half Common.

Figure 49. A comparison of common stock with a 12 percent loan with a 12 percent bond for the heir of a Class V farm who initially owns 40 percent divided half in common and the other half in common, a bond or in a loan.

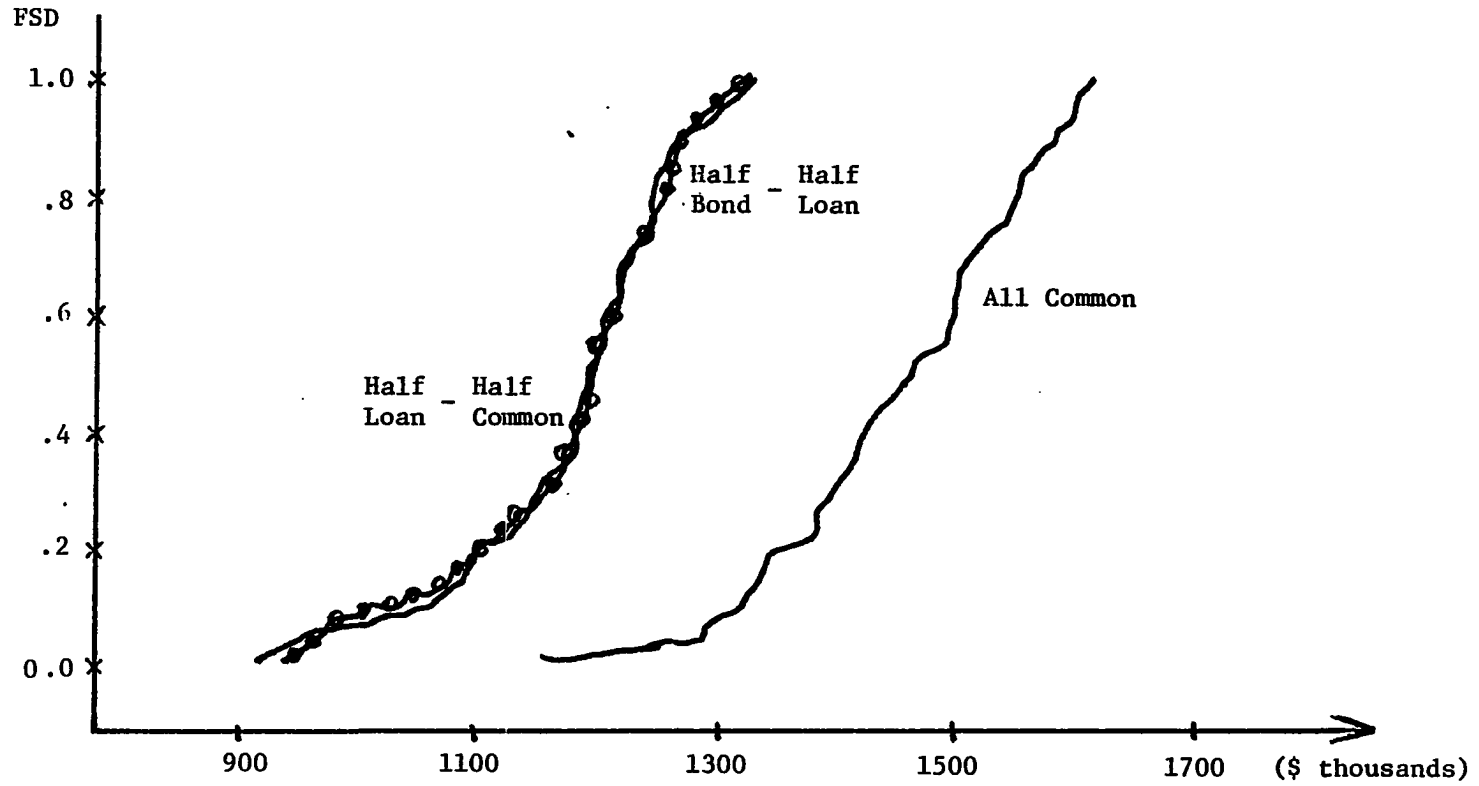


Figure 50. A comparison of a loan with a bond, both at 12 percent, for the parents of a Class IV farm who initially own 80 percent (half in common and half in a bond or a loan) and an heir who initially owns 20 percent all in common

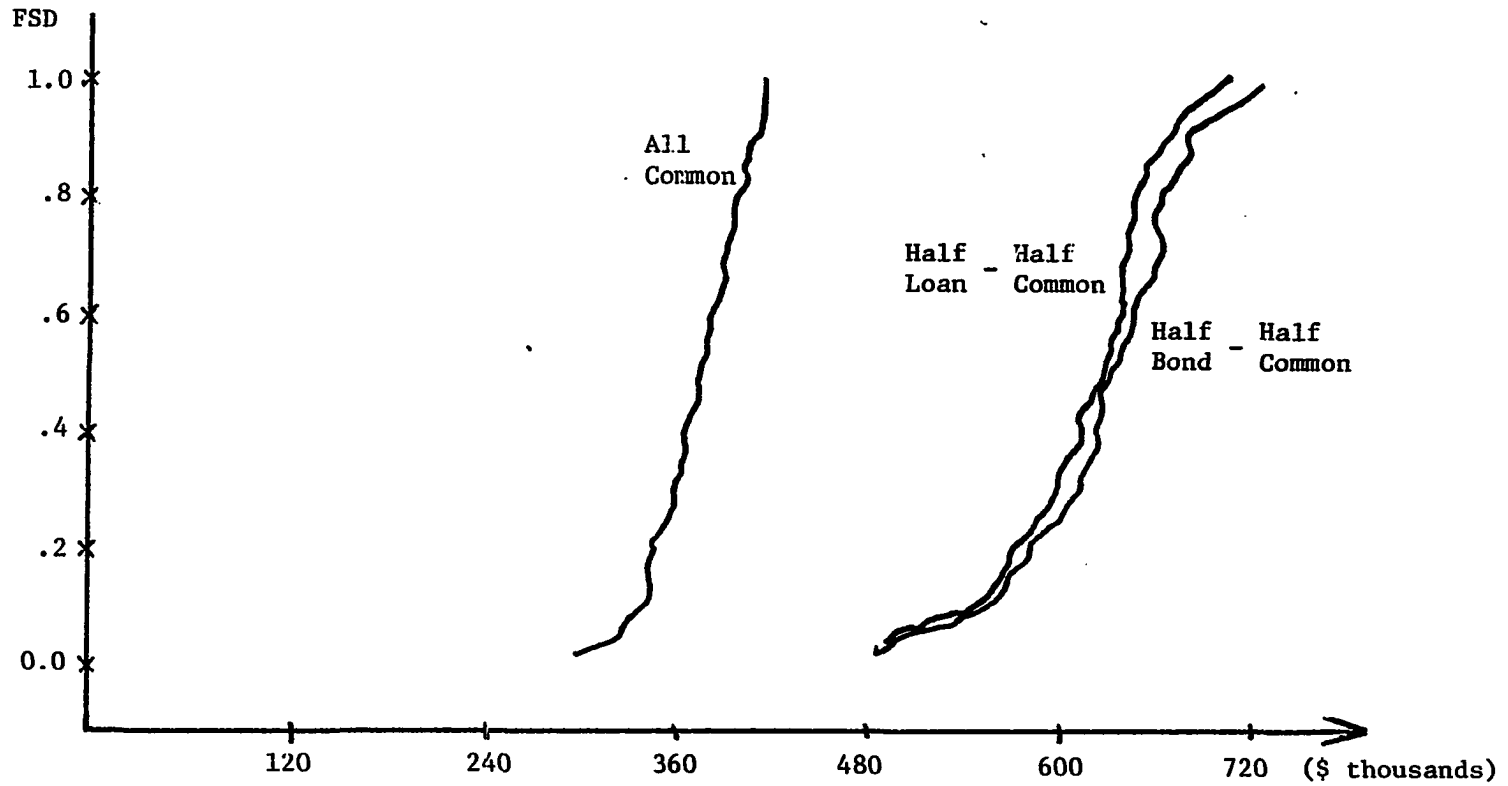


Figure 51. A comparison of a loan with a bond, both at 12 percent, for the heir of a Class IV farm who initially owns 20 percent (all in common stock) and the parents who own 80 percent (half in common stock and half in common, a bond or a loan)

(her) common stock for a ten-year constant principal loan. The two curves cross in Figure 42 so that under first degree stochastic dominance, the parents are indifferent as to whether the heir holds his (her) interest entirely in common stock or half in common stock and half in a loan. Figure 43 measures the second degree stochastic dominance of these two cumulative density functions of Figure 42. Since the Half Loan-Half Common curve lies entirely to the right and below the All Common curve in Figure 43, all parents whose utility function specification possesses a negative second derivative (i.e., parents who are risk averse) will prefer that the heir hold half his (her) interest in a market-rate loan because this results in the greater expected utility for risk averse parents.

There are two reasons why the utility is greater. First, the proportionate ownership of the residual equity of the corporation changes with the conversion. Before redemption the parents own 80 percent of the common stock and the heir owns 20 percent. After redemption, the parents' equity interest has increased from 80 to 89 percent while the heir's equity interest has decreased from 20 to 11 percent. Earnings retained by the corporation combined with appreciated asset values are distributed equally among the outstanding shares. Therefore, after conversion, the parents participate in 89 percent of the increase in corporate value while the heir participates only in 11 percent.

Second, with the issuance of a loan, interest payments on the loan reduce after-tax corporate cash flow. The accumulated income taxes paid by the corporation (for the mean solutions) decrease from \$1,102,033 for

the all common case to \$1,055,291 when the heir converts half his (her) income stock to a loan. However, the after-tax net income of the firm decreases from \$170,165 to \$164,858 with conversion to a loan (mean values for 1985). As a result, the corporation has less after-tax earnings available to distribute. Although the parents have a greater participation rate after conversion, they participate in a smaller cash flow stream. The parents' average annual growth rate in equity increases from 16.14 percent before conversion to 16.23 percent after conversion. In an argument analogous to the earlier discussion of Table 16 and 17, the parents benefit from the reduction in corporate taxes to a slightly greater degree than the reduction in corporate cash flows. However, the Half Loan-Half Common and All Common curves in Figures 42 and 43 are so close together as to suggest that even though stochastic dominance can differentiate between them, no substantial difference exists. From Table D-1 of Appendix D, the mean and variance of the All Common curve in Figure 42 are \$3,781,500 and \$252,506,300,000 whereas the mean and the variance of the Half Loan-Half Common curve are \$3,883,500 and \$285,262,800,000.¹

Figure 44 shows the results of the same financing situation, but for the heir. Under FSD, the child is also indifferent between holding his (her) entire initial interest in common stock and a market-rate loan.

¹If the Half Loan-Half Common and All Common curves of Figures 42 and 43 are assumed to be independent and normally distributed, the null hypothesis that the means are equal can be tested as is done in Appendix D. From Table D-1 of Appendix D there is a better than 80 percent change the means are approximately equal.

However, if the heir is risk averse ([SSD] in Figure 45), then the son will unambiguously prefer the Half Loan-Half Common to All Common situation.¹

The reason why this occurs is the timing at which the heir acquires new shares of common stock. Conversion to an loan requires the heir initially to give up common stock. As the loan is amortized over the planning horizon, the heir is assumed to reinvest the principal (along with interest and any other excess cash flows) in new shares of common stock on an annual basis. For the heir, the mean of the Half Loan-Half Common curve in Figure 44 is larger than the mean of the All Common curve (\$1,085,900 versus \$1,048,200) while the variance is smaller (\$14,280,300,000 versus \$18,796,400,000).

A comparison of the detailed solutions (such as those shown in Tables 16 and 17) at the mean values provides little insight as to why the heir must be risk averse to prefer the Half Loan-Half Common to All Common situation. However, the following simplified example in Table 18 will clarify what is happening. Suppose at time t the heir owns 100 shares of common stock valued at \$100 per share for a total equity interest in the firm of \$10,000. On the first day of the year, the heir converts 50 shares of common stock into a \$5,000, one year, 12 percent loan. After conversion, the heir owns 50 shares of stock valued at \$5,000 and a \$5,000 loan. Further suppose that during the next year

¹The test of the null hypothesis of equal means can be rejected at the 80 percent confidence level (Appendix D, Table D-1).

there are three possible outcomes for the firm. If the firm has a bad year, its after-tax rate of growth in equity is 8 percent. If the year is mediocre, then the average rate of growth in equity is 9.6 percent. And if the firm enjoys an exceptional year, the rate of growth will be 12 percent. The heir receives the loan principal of \$5,000 and interest of \$480 after taxes (assuming the heir's marginal tax rate is 20 percent) regardless of whether the firm has an exceptional year or a bad year. However, the value of the heir's common stock varies with the outcome. As a result, if the firm's after-tax rate of growth in equity is 8 percent, the heir is \$80 better off by converting half the common to a loan. If the rate of growth is 12 percent, the heir is \$120 worse off from the conversion.

There is some after tax rate of growth in equity (in this simple example 9.6 percent) below which the heir is better served by converting to a loan and above which the heir is better served by holding all common stock.

Now to relate this simple example to the curves in Figure 44. Each point on the Half Loan-Half Common curve in Figure 44 measures a change in the net terminal wealth of the heir over the ten-year planning horizon. Points lying to the lower left on the curve represent smaller increases in net worth than do the points lying toward the upper right. Equivalently, points to the lower left correspond to relatively smaller rates of growth in equity while points lying to the upper right correspond to relatively larger rates of growth. Below the rate of growth corresponding to \$1,250,000 in Figure 44, the heir prefers to hold

the loan because the after-tax return on the loan exceeds the after-tax rate of growth in common stock. In Figure 44, this occurs at such a high rate that the probability of realizing a higher rate is relatively small. Therefore, as shown in Figure 45, a risk averter is not willing to gamble on the firm's rate of growth exceeding the fixed return on the loan and the heir prefers the loan under second degree stochastic dominance.

Figures 46, 47, and 48 show the same situation for the parents when the parents initially own 60 percent of the firm and the heir owns 40 percent. When the heir's initial 40 percent common stock position is split equally between common stock and a market-rate loan, the parents are indifferent under FSD (Figure 46) and SSD (Figure 47). Only under third degree stochastic dominance (TSD) do the parents prefer that the heir hold half his (her) interest in a loan (Figure 48). That is, all parents who are nonsatiated in wealth, who are risk averse, and who evidence decreasing absolute risk aversion will unambiguously prefer that the heir hold a market-rate loan in lieu of common stock. However, as shown in Table D-1, there is little substantial difference between the means of the two curves.

The heir also prefers the Half Loan-Half Common arrangement because he (she) is receiving the annual interest and principal payments and can reinvest the proceeds in new common stock (Figure 49).

Figures 50 and 51 compare common stock with a 12 percent constant principal loan when the parents' share is split equally between common stock and a loan. There are several reasons why the parents might consider such a financing strategy as this. For example, if dividends

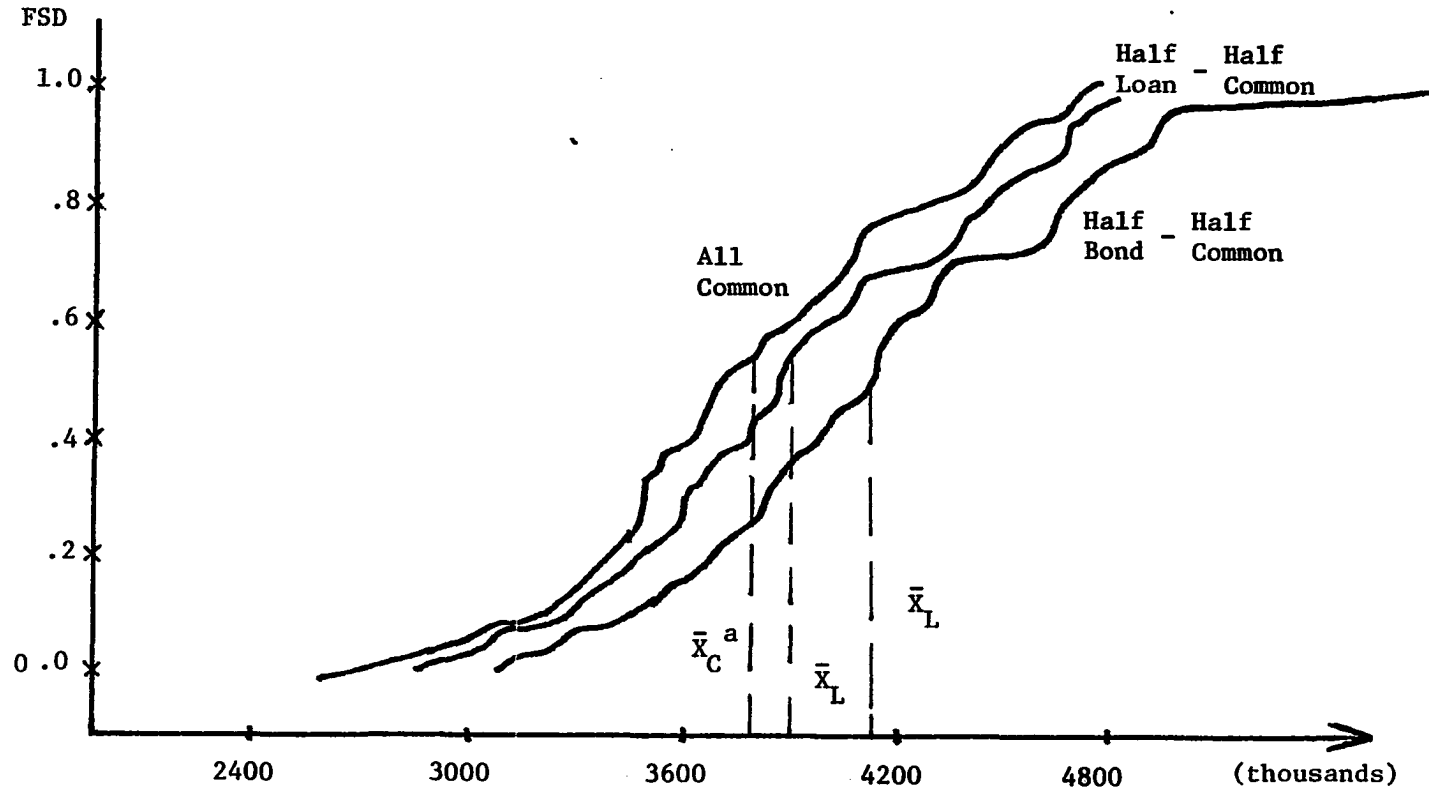
are not being paid and the parents can not satisfy the reasonableness test for salaries or directors' fees, interest payments can be used to support their standard of living. Another possible reason is the sense of assurance that interest payments provide as a known source of income. Finally, common stock with voting rights carries with it responsibility to direct and manage the firm. When parents reach a certain stage in their lives, they may no longer wish to bear that responsibility. In Figure 50, the parents in a Class IV farm who initially own 80 percent and convert half their common stock to a 12 percent loan, prefer, under first degree stochastic dominance, the All Common outcome to the Half Common-Half Loan. The heir prefers that the parents own half their interest in common and half in a loan under first degree stochastic dominance (Figure 51).

The reason that the parents prefer to keep all their interest in common instead of half in a loan is that their proportionate interest in the equity decreases from 80 percent to $(80-40)/(100-40)$ or 67 percent. Although the parents receive periodic interest payments which can be reinvested in the firm, this is more than offset by the decrease in their equity participation. The heir prefers that the parents hold a loan because his proportionate equity interest increases from twenty percent to one-third. This more than offsets the decrease in the cash flow stream from the periodic interest payments. Although only the results for a class IV farm are presented here, the same preference ordering is true for the other representative farm sizes.

Common Stock Versus a Constant Principal Loan
at Below Market Rate of Interest

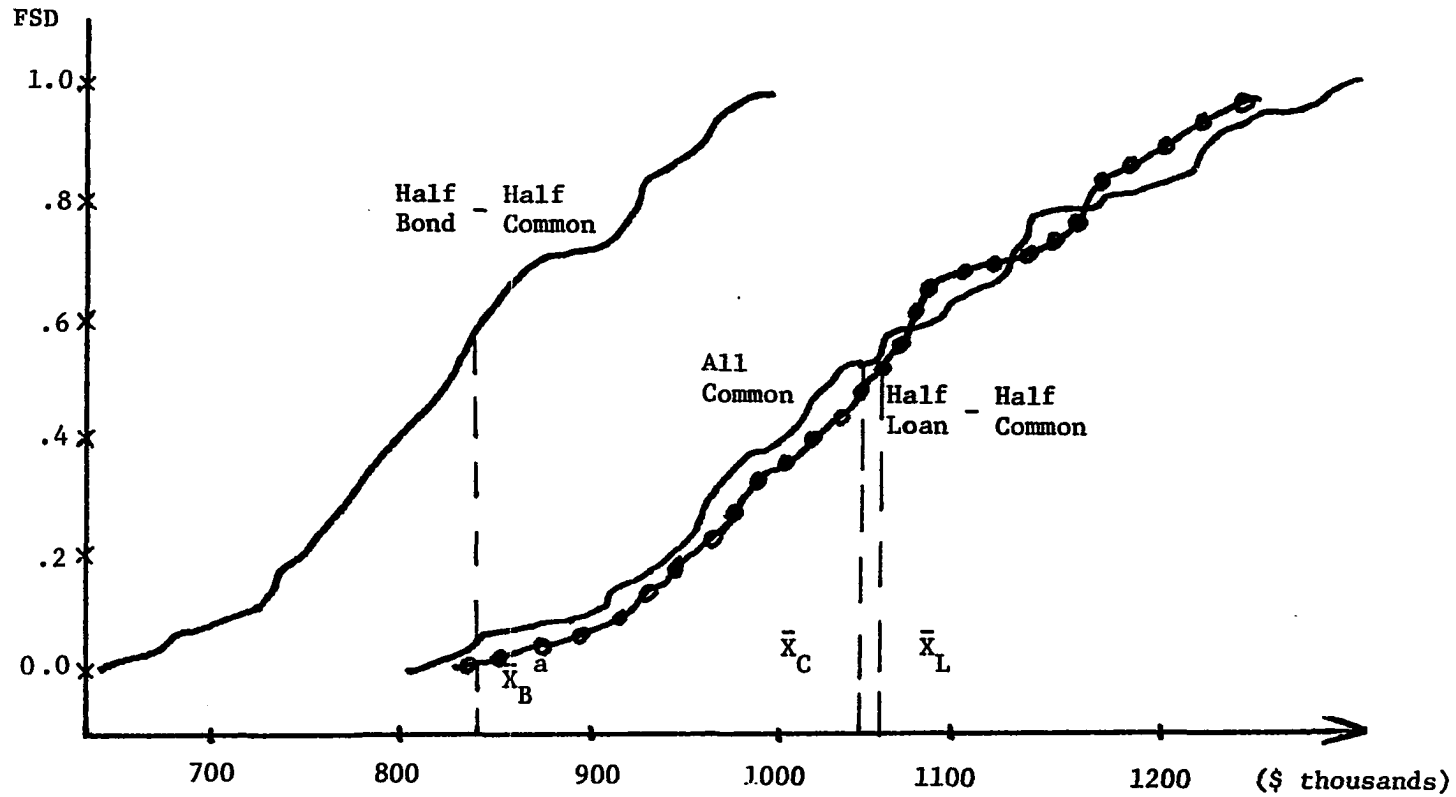
Figures 52 to 54 show the tradeoffs for the parents and the heir for a Class V farm initially divided 80/20 when the heir redeems half his (her) common stock for a below market-rate, ten-year constant principal loan (assumed to be 9 percent). In Figure 52, the parents in this situation prefer the heir to hold half his (her) interest in a loan under first degree stochastic dominance. However, the means for the Half Loan-Half Common curve and the All Common curve in Figure 52 are not significantly different at the 80 percent level of confidence (\$3,924,400 as opposed to \$3,781,500 with variances of \$287,296,000,000 for the Half Loan-Half Common and \$252,506,300,000 for the All Common financing arrangement). The heir, however, does not care whether he (she) has an all common interest or a half common-half below-market loan under first degree stochastic dominance, as shown in Figure 53. The mean of the Half Loan-Half Common curve in Figure 53 is slightly larger (\$1,059,000 versus \$1,048,200) and the variance is slightly smaller (\$13,689,000,000 versus \$18,796,000,000) but the mean difference is not significant. Therefore, there is little substantial difference for the heir between the two financing arrangements. If the heir is risk averse, he (she) will unambiguously prefer the Half Loan-Half Common to the All Common situation under second degree stochastic dominance (Figure 54).

The use of a constant principal loan by the heir instead of common stock affects the cash flows of the firm and the rate of appreciation per



^a Mean values: \bar{X}_C is the mean of All Common, \bar{X}_L is the mean of Half Loan-Half Common, and \bar{X}_B is the mean of Half Bond-Half Common.

Figure 52. A comparison of common stock with a 9 percent loan with a 9 percent bond for the parents of a Class V farm who initially own 80 percent (all in common stock) and an heir who initially owns 20 percent (half in common and the other half in either common, a loan or a bond)



^a Mean values: \bar{X}_C is the mean of All Common, \bar{X}_L is the mean of Half Loan-Half Common, and \bar{X}_B is the mean of Half Bond-Half Common.

Figure 53. A comparison of common stock with a 9 percent loan with a 9 percent bond for the heir of a Class V farm who initially owns 20 percent divided half in common and the other half in either common, a loan or a bond

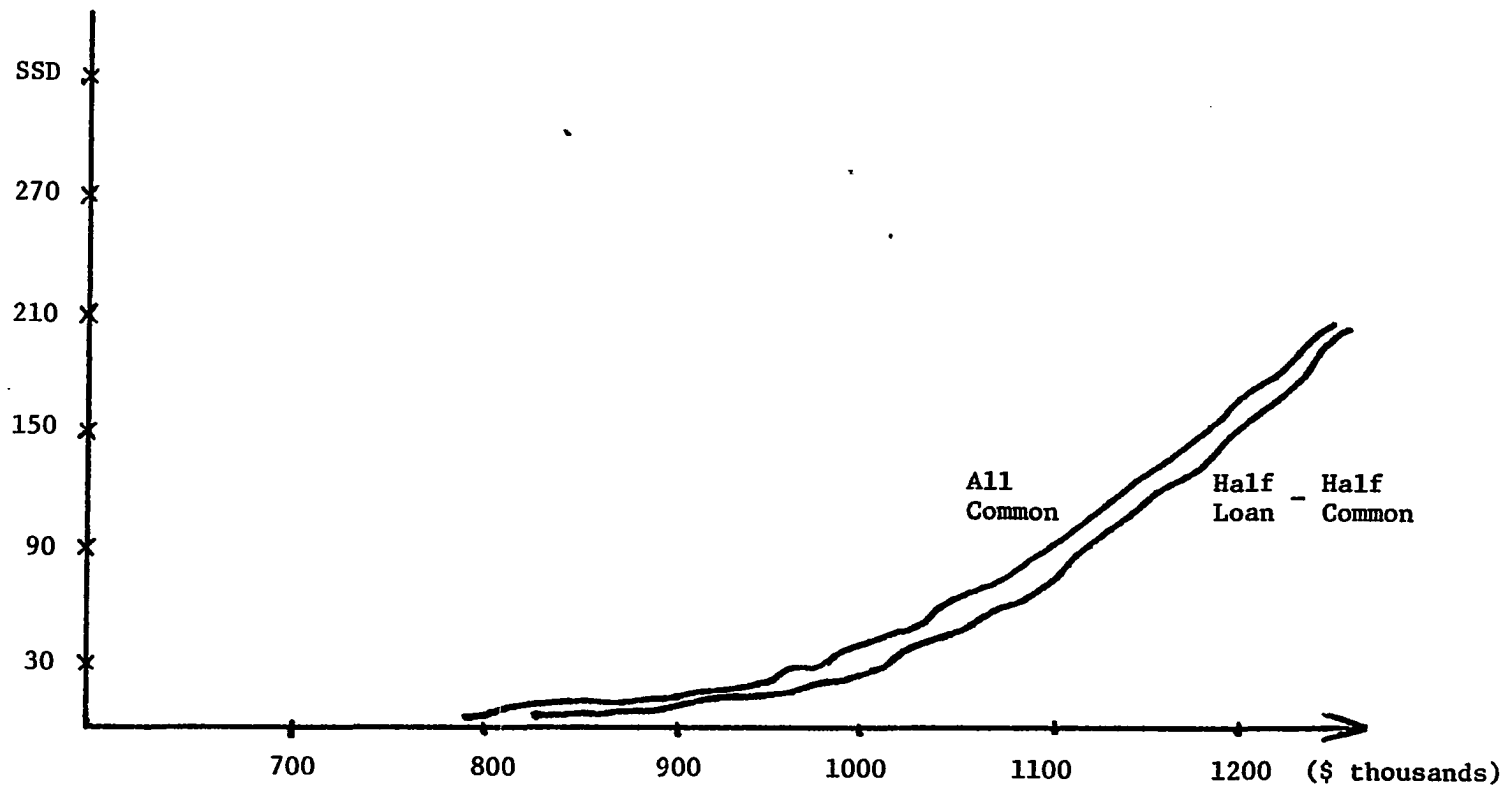


Figure 54. Second degree stochastic dominance of the situation shown in Figure 53

share of common stock. Before conversion, the organization paid accumulated income taxes of \$1,102,033 while after conversion, it paid \$1,067,611 (for the mean solutions). However, the \$5,752 interest payment for 1985 to the heir reduced the firm's 1985 after-tax net income from \$170,165 before conversion to \$166,281 after conversion. The average annual growth rate in equity increased with the conversion from 16.14 percent to 16.33 percent for the parents and from 17.03 percent to 24.01 percent for the heir. The attractiveness of the loan to the parents or the heir is a function of the after-tax cost of debt servicing to the firm, the after-tax rate of return on the loan, and the rate at which the per-share value of the common stock is increasing. The interactions of these three will determine whether the loan or the common stock is more desirable.

Common Stock Versus a Market Rate Bond

If the loan discussed in the previous two sections did not require periodic principal payments but merely a balloon payment at maturity, it would be, in effect, a ten-year bond between the heir and the corporation. The Half Bond-Half Common curves in Figures 42 through 49 represent the distribution of terminal wealth of the parents and the heir when the heir holds half his (her) interest in common stock and half in a ten year bond paying the market rate of interest.

In all cases, the parents prefer the redemption of half the heir's common stock for the bond under FSD. Conversely, the heir prefers not to hold the bond in all cases under FSD. The parents prefer that the heir

redeem his (her) common stock for a bond because this increases the parent's participation in the growth of net worth by more than the decrease in the after-tax corporate cash flow. Accumulated corporate income taxes decline from \$1,102,128 to \$949,959 because of the tax deductible interest payments on the bond. However, the after-tax net income of the firm declines from \$170,175 to \$151,905 because of the interest payment to the heir (mean values for 1985 for the parents and heir of a class V farm divided 60/40, as shown in Figure 46 and Figure 49). After conversion to a bond, the parents' average annual equity growth rate increases from 16.43 percent to 17.63 percent while the heir's annual growth rate declines from 16.16 to 15.94 percent. As can be seen in Table D-1, all comparisons between common stock and a market rate bond are significantly different at the 95 percent confidence level. Intrafamily conflict occurs with the proposed use of bonds bearing the market rate of interest.

However, if the parents' objective function of maximizing their expected utility of terminal wealth is constrained by a desire to facilitate the transfer of the farm to the heir, then bonds will never be employed as a transfer device because the heir's expected utility of wealth is always higher when all of the heir's interest is in common stock than when that interest is split between common stock and a bond.

Common Stock Versus a Below-Market Rate Bond

The Half Bond-Half Common curves in Figures 52 through 54 represent the situations where the heir's interest is divided into half common

stock and half in a 9 percent bond, where 9 percent is three percent below market. In all cases, the results are the same as for the 12 percent bond. That is, the heir is worse off in terms of terminal wealth with the use of the bond, and mean differences are significantly different at the 95 percent confidence level. This supports the argument of the previous section in that the bond with either a market or below market rate of interest is not effective in facilitating intergenerational transfers.

However, if the effects of the interest rate are symmetrical, these results lead to the hypothesis that a bond with an above market rate of interest would improve the heir's economic position relative to the full ownership of common stock. Therefore, parents who are genuinely concerned about transfer of the farm to the heir should consider bonds only if the bond carries an above-market interest rate.¹

Constant Principal Loans Versus Bonds

Figures 42 through 54 can be used to compare situations where the heir's interest in the corporation is composed half of stock and half in either a constant principal loan or half in a bond. Figures 42 through 49 compare the situation when both the bond and loan carry a market-rate of interest. Figures 52 through 54 compare the two at 9 percent.

¹This is not as attractive an option with nonfarm heirs. The on-farm heir is assumed to reinvest any excess personal cash flow in the corporation whereas the nonfarm heirs would not be expected to do so. Therefore, with an on-farm heir, the corporation does not experience the cash flow problems it would with nonfarm heirs.

In all cases, the constant principal loan is preferred to the bond by the heir, while the parents always prefer the bond over the constant principal loan.¹ Referring to Appendix D, all mean differences between Half Loan-Half Common and Half Bond-Half Common for the parents and the heir are significant at the 80 percent level of confidence and some are significant at the 95 percent level. The reason for this preference scheme is the role that periodic principal payments play. The heir receives interest on the loan balance in each case but with the loan, the heir also receives periodic principal payments which can be used to purchase new shares of common stock. As a result, with a loan, the heir receives annual income and enjoys the right to increase his (her) proportionate ownership of the firm. Conversely, the parents' proportionate equity in the firm is continually being diluted due to the issuance of new shares to the heir. When the bond arrangement is used, the principal of the bond is not used to acquire new common stock, and the rate of dilution of the parents' interest in the corporation is not as rapid.

However, since interest is paid on the outstanding balance, the decrease in the after-tax cash flow stream of the corporation is greater with the bond than with the loan. The two effects partially offset each other, but the loss in participation through dilution is greater than the

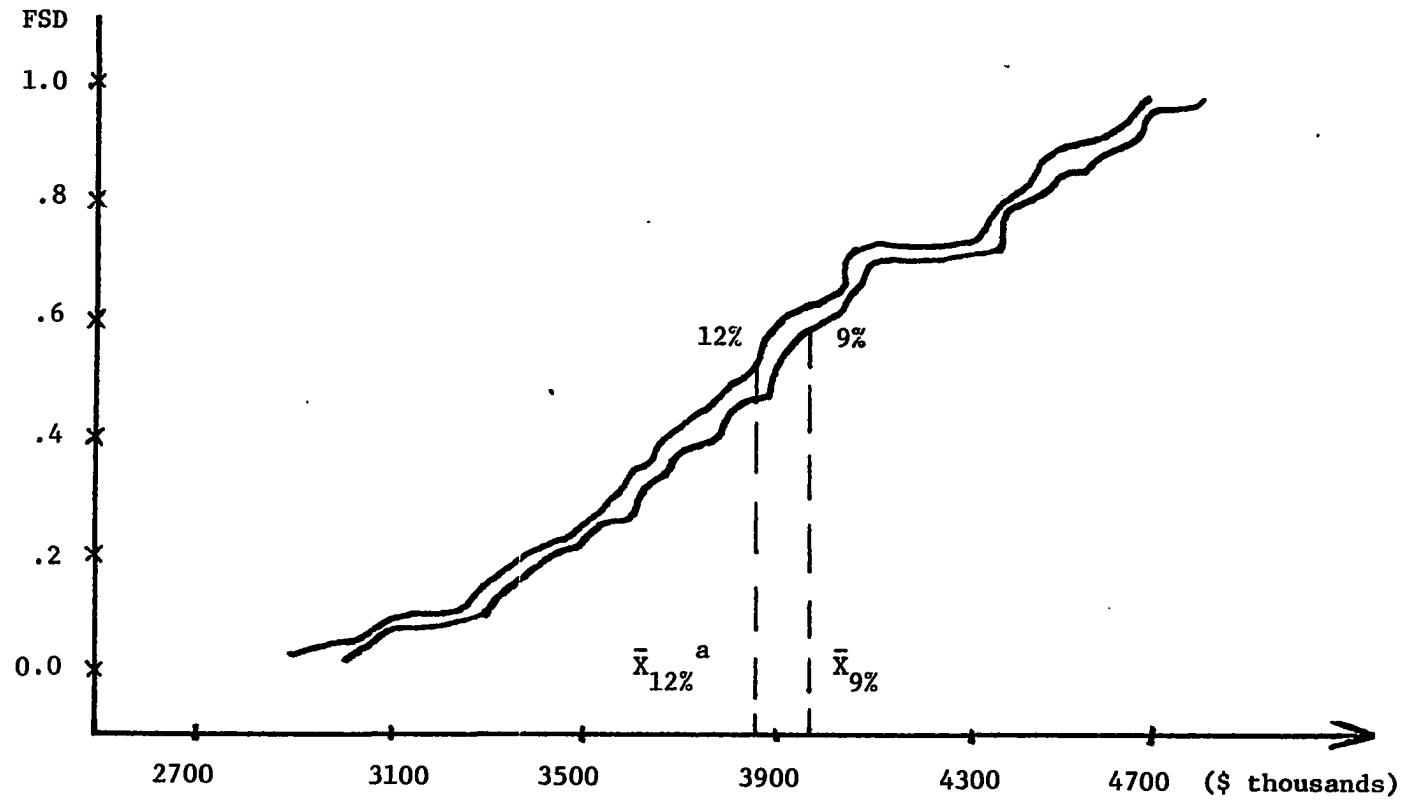
¹As discussed in the earlier sections, the parents prefer the exclusive use of common stock, but the heir prefers either a bond (held by the parents) or a loan (held by the heir). The parents ranking of preferences when they have the option of converting half their common stock into a loan or bond would be common stock over either a bond or a last. Whereas, the heir prefers the bond over the loan over all common stock.

cash flow difference from the interest payments, so the parents prefer the bond to the loan.

If the parents' objective function includes the desire to transfer the farm to the heir, it is preferable to use the constant principal loan over the interest-only bond.

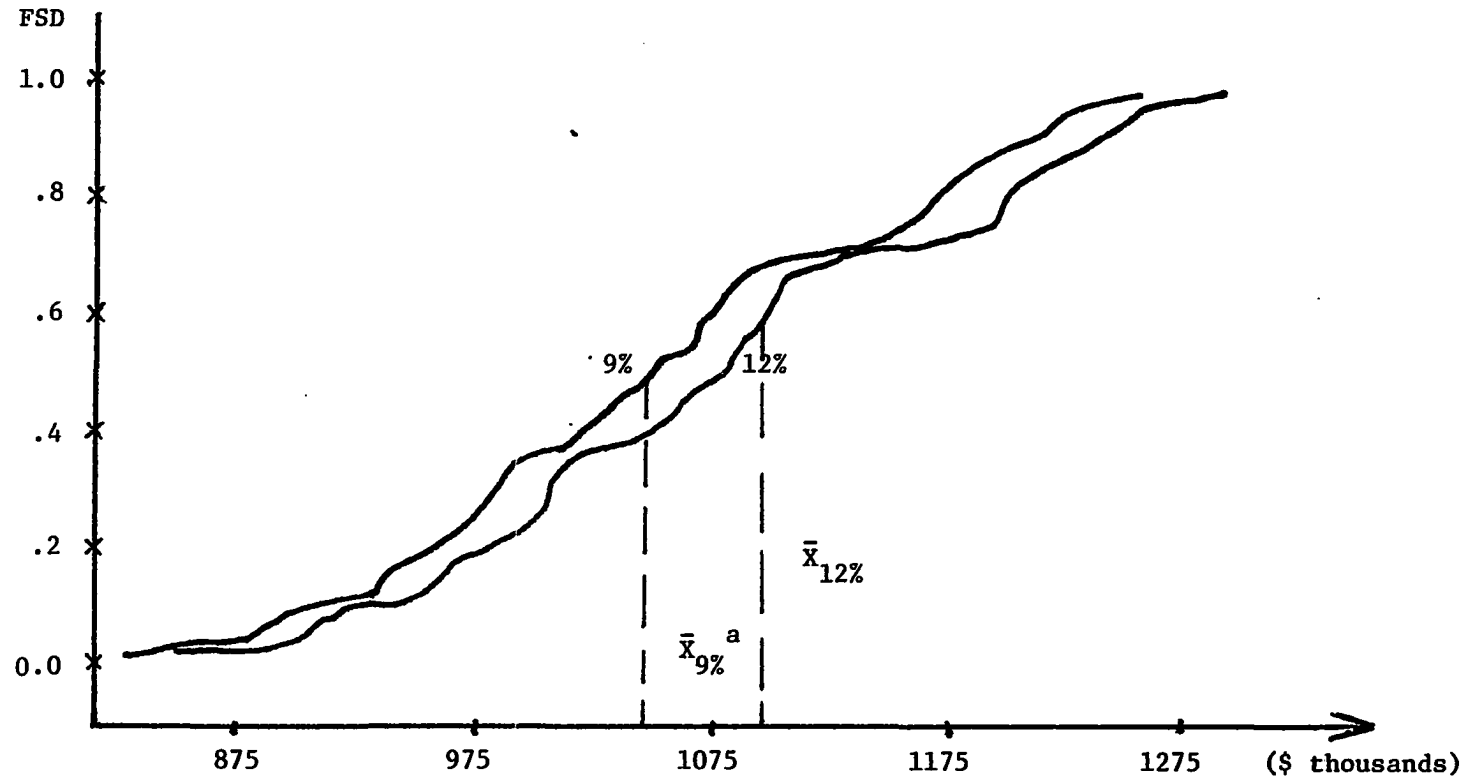
Market Versus Below Market Interest Rates

Figures 55 through 58 compare a below-market constant principal loan against a market-rate constant principal loan. Figures 59 through 60 make the same comparison for bonds. In all situations the parents prefer the lower interest rate while the heir prefers the higher rate under FSD. For a class V farm initially owned 80 percent by the parents, and with the heir's conversion of half his (her) common stock to a loan, the parents' average annual equity growth rate is slightly larger when 9 percent interest is paid on the loan than when 12 percent is paid. Conversely, the heir's average annual growth rate in equity increases slightly when the interest rate on the loan is increased from 9 to 12 percent. Similarly, when the parents' interest is divided half in common stock and half in either a bond or a loan, the parents prefer 12 percent to 9 percent under first degree stochastic dominance. At the same time, the heir prefers, by first degree stochastic dominance, that the parents receive the below-market interest rate. However, as can be seen in Table D-1, the means are not significantly different at the 80 percent level of confidence (with the exception of Figure 60).



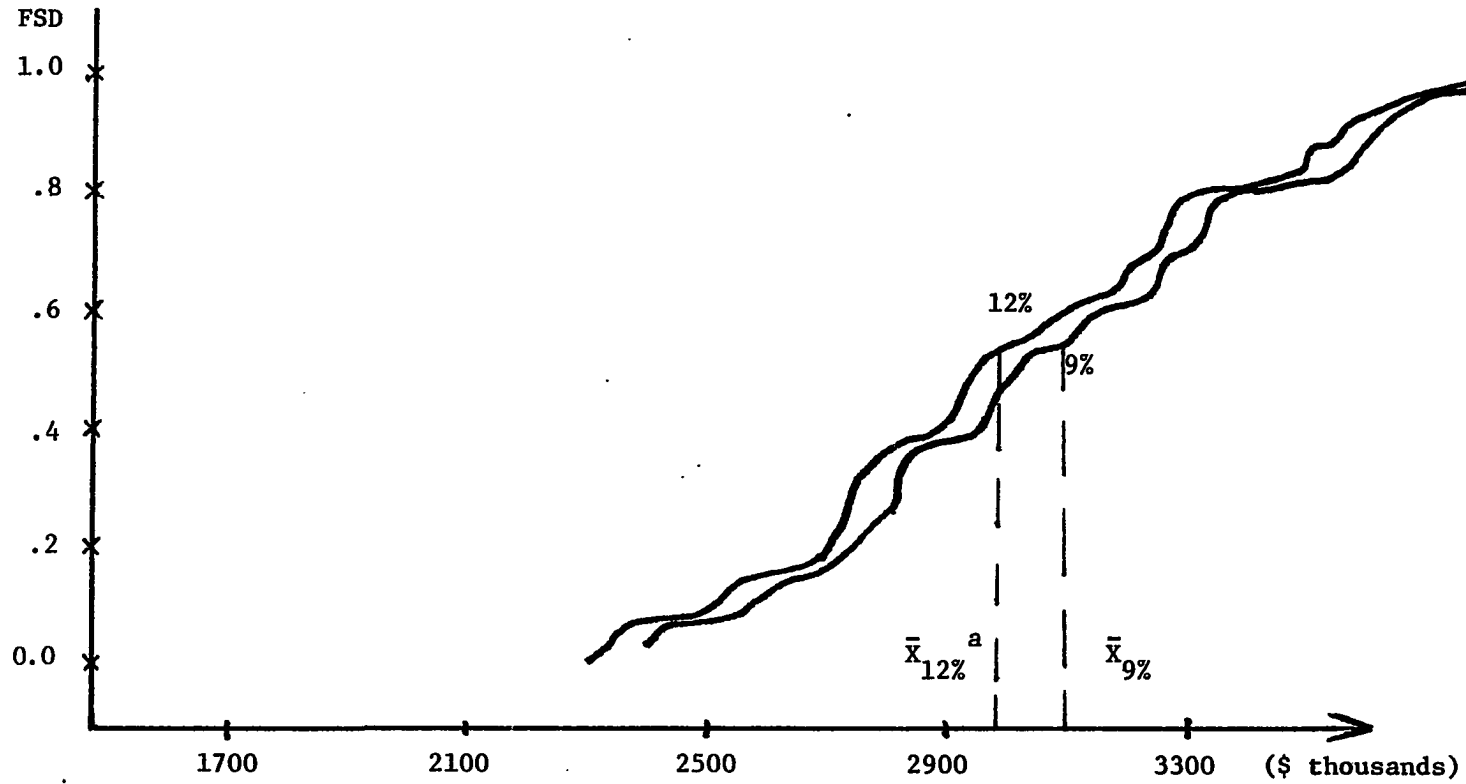
^a Mean values.

Figure 55. A comparison of a 12 percent and a 9 percent loan for the parents of a Class V farm who initially own 80 percent (all in common stock)



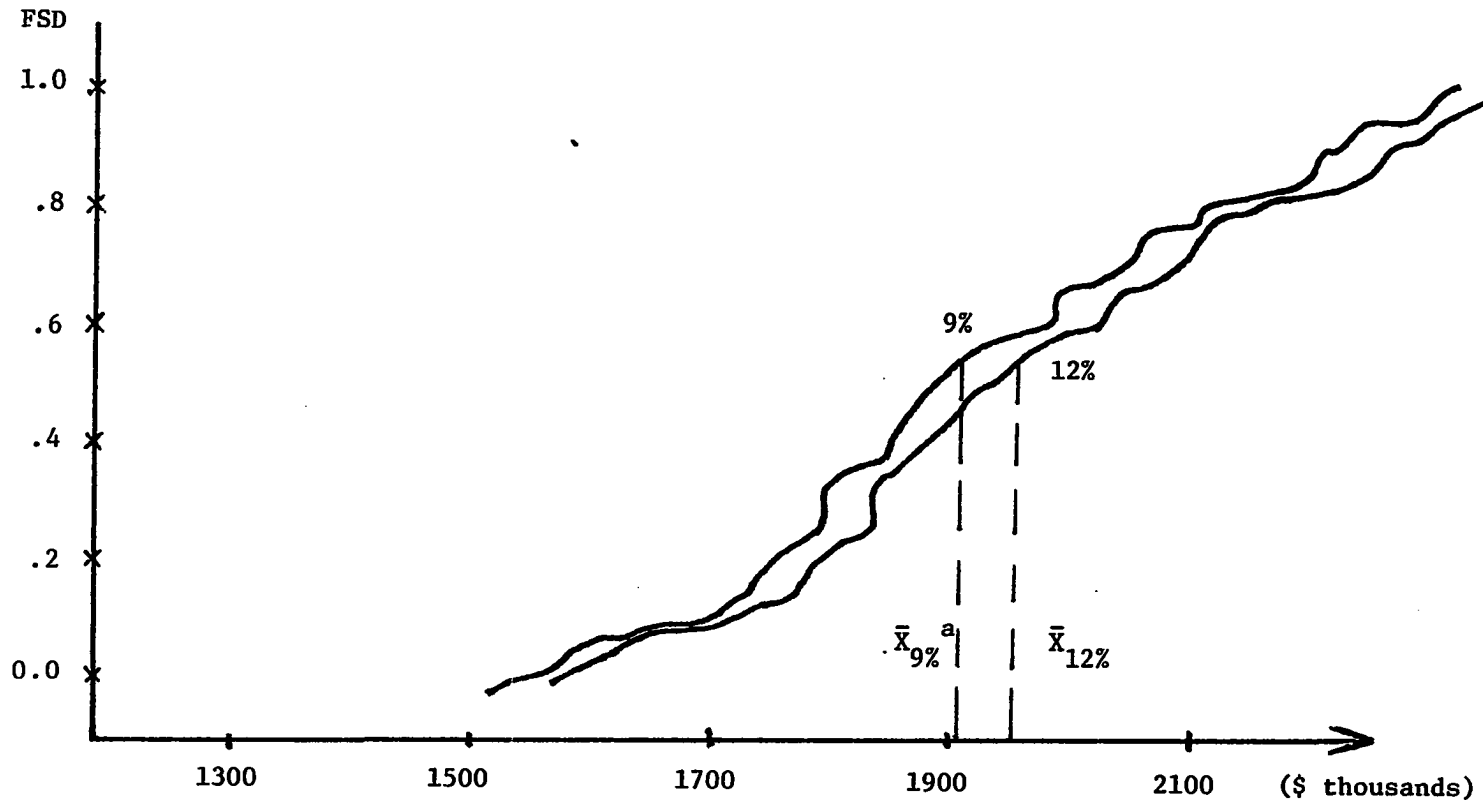
^a Mean values.

Figure 56. A comparison of a 9 percent and a 12 percent loan for the heir of a Class V farm who initially owns 20 percent (half in common stock and half in a loan)



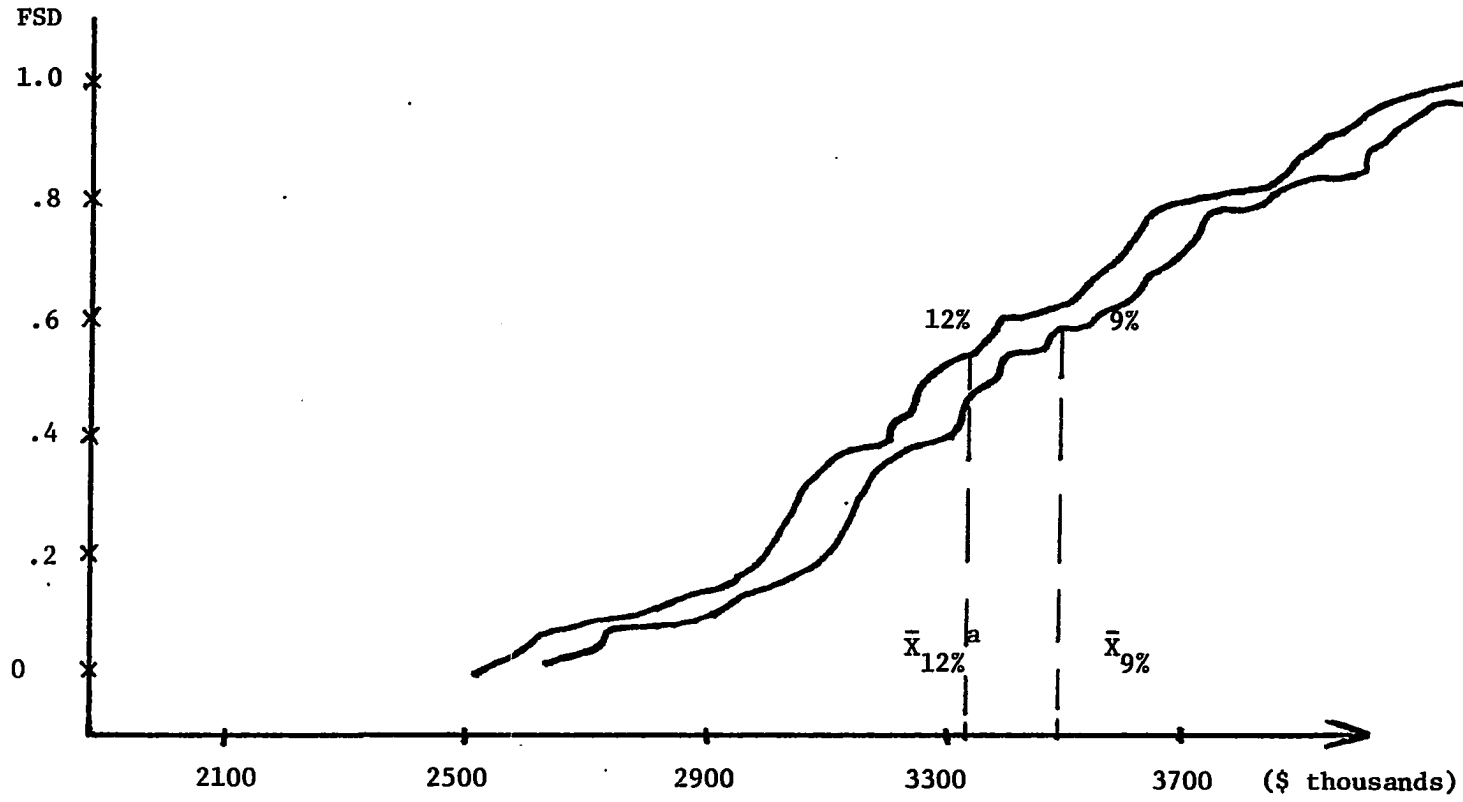
^a Mean values.

Figure 57. A comparison of a 9 percent and a 12 percent loan for the parents of a Class V farm who initially own 60 percent (all in common stock)



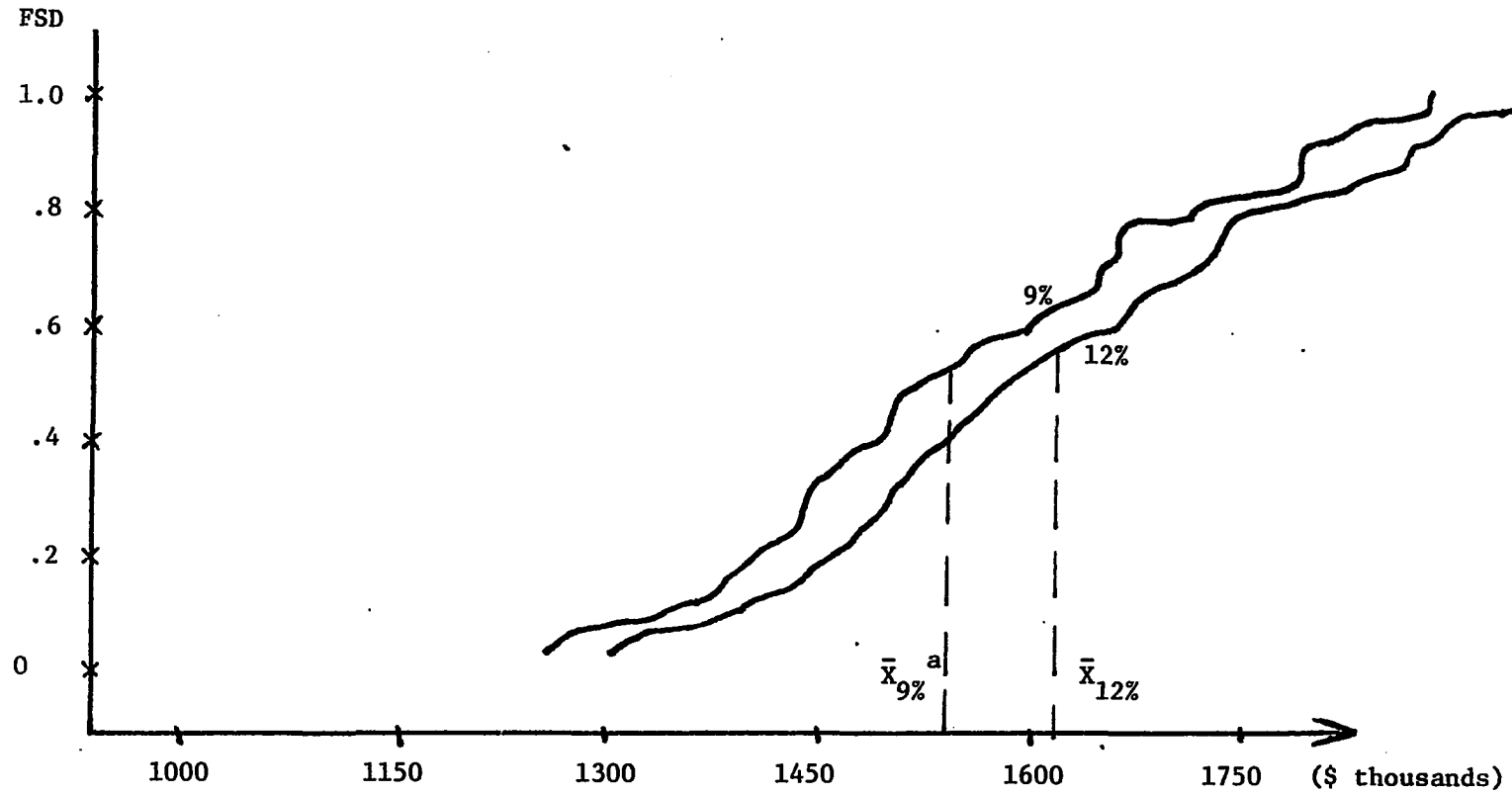
^a Mean values.

Figure 58. A comparison of a 9 percent and a 12 percent loan for the heir of a Class V farm who initially owns 40 percent (half in common and half in a loan)



^a Mean values.

Figure 59. A comparison of a 9 percent and a 12 percent bond for the parents of a Class V farm who initially own 60 percent (all in common stock)



249a

^a Mean values.

Figure 60. A comparison of a 9 percent and a 12 percent bond for the heir of a Class V farm who initially owns 40 percent (half in common stock and half in a loan)

In terms of the maximization of expected utility, the two parties will be in conflict over the relevant interest rate to use. If the intergenerational transfer is desired, the process will be facilitated by placing an above market rate of interest on debt issued to the heir and a below market rate of interest on debt issued to the parents.

CHAPTER VII. SUMMARY AND CONCLUSIONS

The empirical findings of selected financing arrangements were discussed and presented in Chapter VI. The financing arrangements studied were the mix of ownership between the parents and the heir, the use of salaries and directors' fees as opposed to dividends, and the redemption of common stock for either a market or below-market rate loan of bond by the parents or the heir. Those findings are summarized below.

Mix of ownership between parents and heir

Three ownership mixes were analyzed as starting points for later comparisons. These were 100 percent owned by the parents (100/0), 80 percent owned by the parents and 20 percent owned by the heir (80/20), and 60 percent owned by the parents and 40 percent owned by the heir (60/40).

The analysis shows that for all representative farms, the orders of preference for the parents and the heir is:

for the parents: $100/0 >_1 80/20 >_1 60/40$;

for the heir: $60/40 >_1 80/20 >_1 100/0$.¹

These orderings highlight the intrafamily conflict which arises whenever parents attempting to maximize their expected utility of wealth

¹The notation $A > B_i$ is interpreted A is unanimously preferred to B by ith degree stochastic dominance. The notation $A \not> B_i$ is interpreted A is not preferred to B nor B to A by ith degree stochastic dominance.

wish to initiate the intergenerational transfer during life. For parents to initiate an intergenerational transfer process during their lifetimes, their objective of maximizing their expected utility of wealth will have to be constrained by the desire to continue the farming operation across generations. If it is not, the parents will never give away any of the firm and will retain as much control over the farm as they can for as long as they can.

Income sharing plans

Common methods of sharing income between the parents and the heir are through the use of salaries, directors' fees, and dividends on common stock. For all situations analyzed, it was true that the ordering preference for the parents and the heir is:

salaries and directors' fees $>_1$ dividends.

This is because salaries and directors' fees are distributed from before-tax earnings of the firm whereas dividends are distributed from after-tax earnings.

Redemption by the heir of half his (her) common stock for either a loan or a bond

When the loan or the bond is at the market or below market rate of interest, the ordering preferences of the parents and the heir for selected initial ownership mixes are those shown in Table 18.

In all situations, the parents prefer that the heir convert half his (her) commonstock into a bond because the parents' increased

Table 18. Ordering preferences for the parents and the heir when the heir converts half his (her) equity interest into a bond or loan*

<u>For the</u>	When the parents initially own:	And the heir initially owns:	And the interest rate is:	The ordering preference is:
PARENTS	80%	20%	12%	Bond $>_1^a$ Loan $>_2^b$ All Common
			9%	Bond $>_1$ Loan $>_1$ All Common
	60%	40%	12%	Bond $>_1$ Loan $>_3^c$ All Common
			9%	Bond $>_1$ Loan $>_1$ All Common
HEIR	80%	20%	12%	Loan $>_2$ All Common $>_1$ Bond
			9%	Loan $>_2$ All Common $>_1$ Bond
	60%	40%	12%	Loan $>_1$ All Common $>_1$ Bond
			9%	Loan $>_2$ All Common $>_1$ Bond

*Numerical values (means, variance, skewness, and kurtosis) that accompany this table can be found in Table D-1, Appendix D.

$^a>_1$ Means unaminously preferred by first degree stochastic dominance.

$^b>_2$ Means unaminously preferred by second degree stochastic dominance.

$^c>_3$ Means unaminously preferred by third degree stochastic dominance.

participation in the after-tax cash flows of the firm more than offsets the decrease in the size of the cash flow stream caused by the interest payments. For diametrically opposed reasons, the heir least prefers the bond.

The parents also prefer that the heir convert half his (her) common stock into a loan. The preference is not as strong as with the bond. (In fact, the parents prefer that the heir hold a bond rather than a loan in all situations.) This is because the periodic principal payments made to the heir are reinvested in new common stock which continually erodes the parents' rate of equity participation in after tax cash flows.

The heir prefers to convert half his (her) common stock to a loan because their after-tax return on the loan exceeds the rate of return foregone in participating in the growth in the equity of the firm.

If the parents and the heir are risk averters, they both can benefit from the heir's holding of a loan. Since there is no interpersonal conflict in this situation, creation of an intrafamily loan to the heir increases the expected utility of both the parents and the heir.

The same is not true when considering a bond. The parents benefit while the heir loses. In this situation, intrafamily conflict exists, and the outcomes can't be predicted. However, if the parents wish to facilitate the intergenerational transfer of the farm to the heir, the issuance of a bond to the heir has the exact opposite effect. That is, it transfers wealth to the parents from the heir.

Altering the interest rate from 12 percent to 9 percent does not significantly alter the ordering preferences. However, an increase in

the interest rate decreases the after-tax cash flow stream of the firm. As the interest rate is increased, the heir's income stream increases at the expense of the parents. Intrafamily conflict occurs and it is not possible to ascertain the interest rate which will maximize the combined utilities of the parents and the heir.

Suggestions for Further Research

The study discussed one part of the intergenerational transfer process--namely, the inter vivos effects of a financing arrangement on net terminal wealth. The estate tax consequences were implicitly assumed to be unaffected by the financing arrangement selected. Such is not the case. Special use valuation and the unlimited marital deduction are just two of the estate tax provisions affected by inter vivos financing arrangements. Further research needs to be done in expanding this study to incorporate estate planning and the estate tax consequences.

This study left unaddressed intergenerational transfers complicated by the presence of one or more off-farm heirs. The on-farm heir was explicitly assumed to reinvest in the farm at every available opportunity. To impose this same assumption upon the behavior of off-farm heirs, is undoubtedly unrealistic. As a result, the off-farm heirs would create equity and liquidity drains on the firm that the on-farm heir does not. Financing arrangements that work for the on-farm heir will probably not work at all well for the off-farm heir. Farm families with both on-farm and off-farm heirs will encounter difficulties in transferring the farm to the on-farm heir while simultaneously providing equitable

treatment of the off-farm heirs. Furthermore, many off-farm heirs have little or no desire to own corporate farm stock that doesn't pay dividends nor has a secondary market for resale. These off-farm heirs will want their potential inheritances in liquid form. Much additional thought needs to be devoted to facilitating the intergenerational transfer of the farm while accommodating the needs of the off-farm heirs.

Further research needs to be done in developing multi-member family objective function. This study could identify conflicts between the parents and the heir, but it could not weigh the trade-off of one party's gain against another's loss.

In this study, conversion from equity to debt was permitted only once at the beginning of the planning horizon. In reality, the passage of time alters the parents' and heir's needs and circumstances. Further research should accommodate organizational conversions at times other than the beginning of the planning horizon.

This study relied on the corporate organizational form throughout. The majority of farms are organized as sole proprietorships or informal partnerships. Further research needs to be done on facilitating intergenerational transfers under organizational forms other than the corporation.

Other topics for further research include: more than one class of common stock, debt instruments convertible into equity, variable interest rates, and interest rates indexed to inflation.

Implications

Farm families interested in transferring the farm to the on-farm heir can enhance such transfers through the use of intrafamily loans to the heir bearing the market rate of interest. During inflationary periods of economic activity, the expected utility of the heir is greater with a combination of common stock and constant principal loan than with all common stock. At the same time, the parent's expected utility also is increased when the heir converts half the common stock to an investor interest in a loan. Over the planning horizon, both the heir's and parent's terminal wealth is increased with the loan to the heir.

The same can not be said for the use of a bond. In all situations analyzed, the heir was worse off at the end of the planning horizon in terms of expected utility of wealth when the heir converted half his (her) common stock into a bond. The parent's expected utility increased when the heir held a bond. The conversion of the heir's common stock to a bond increased the wealth of the parents at the expense of the heir. This is counterproductive to facilitating an intergenerational transfer. As a result, the use of intrafamily bonds to the heir should be discouraged when one of the family's objectives is to facilitate the transfer of the farm to the heir.

This study evaluated the effectiveness of selected financing arrangements in facilitating intergenerational transfers. Certain financing arrangements were consistent with intergenerational transfers (such as issuance of a loan to the heir) since both the parents and the heir benefitted and no interpersonal conflicts arose. Other financing

arrangements, however, resulted in interpersonal conflicts with one party benefitting at the expense of the other. Although it was possible to identify that such conflicts arose, this study was not able to resolve such conflicts. What is needed is a family multi-member objective function which can weight one member's loss in utility against another's gain to determine if the family gains or losses.

This study focused on the during-life effects selected financing arrangements have on intergenerational transfers. The study did not integrate the during-life financial consequence of a particular arrangement with its estate tax consequences. To truly facilitate the intergenerational transfer process, both during-life and death consequences should be evaluated as interdependent parts of a single transfer plan.

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APPENDIX A:

PROOFS OF STOCHASTIC DOMINANCE THEOREMS

The theorems of first, second and third degree stochastic dominance presented in Chapter 3 are reproduced below.

$$\begin{aligned} \text{First degree: } F >_1 G \text{ if and only if } G(x) \geq F(x) & \quad (\text{A-1}) \\ & \text{for all } x \in I. \end{aligned}$$

$$\begin{aligned} \text{Second degree: } F >_2 G \text{ if and only if } G^2(x) \geq F^2(x) & \quad (\text{A-2}) \\ & \text{for all } x \in I. \end{aligned}$$

$$\begin{aligned} \text{Third degree: } F >_3 G \text{ if and only if} & \quad (\text{A-3}) \\ \text{a) } \mu_F \geq \mu_G \text{ and} & \\ \text{b) } G^3(x) \geq F^3(x) \text{ for all } x \in I. & \end{aligned}$$

where $>_1$ denotes "at least as preferred as" for stochastic dominance of degree 1 and F and G are two unequal distributions corresponding to two risky prospects. Each degree of stochastic dominance provides a basis for unanimous preference of one risky prospect over another for all decision makers whose utility functions satisfy the subclass of utility functions defined below.

$$\begin{aligned} U_1 = \{u: u, u' \text{ is continuous and bounded on } I, \\ u > 0 \text{ on } I^\circ\} & \quad (\text{A-4}) \end{aligned}$$

$$\begin{aligned} U_2 = \{u: U_1, u'' \text{ is continuous and bounded on } I, \\ u'' < 0 \text{ on } I^\circ\} & \quad (\text{A-5}) \end{aligned}$$

$$\begin{aligned} U_3 = \{u: U_2, u''' \text{ is continuous and bounded on } I, \\ u''' > 0 \text{ on } I^\circ\}^1 & \quad (\text{A-6}) \end{aligned}$$

For all utility functions of class 1, one and only one of the following will be true when comparing two risky prospects with unequal

¹The interval, I , is of the form (a, b) or (a, ∞) while the interval, I° , represents the interior of I or (a, b) or (a, ∞) .

distributions F and G : 1) $F >_1 G$, 2) $G >_1 F$, or 3) neither $F >_1 G$ nor $G >_1 F$.

The theorems A-1 through A-3 give conditions that are both necessary and sufficient for stochastic dominance. The proof (134, p. 72-77) begins by providing the sufficiency part. First, consider the case where F is discrete with a finite number of jumps at $x_0, x_1, x_2, \dots, x_n$, where $0 = x_0 < x_1 < x_2 < \dots < x_n < 1$ and $f(0) = 0$ and $f(1) = 1$, then the expected value of utility is:

$$E_F[u(x)] = F(x_0)u(x_0) + \sum_{i=1}^n [F(x_i) - F(x_{i-1})]u(x_i) + [1 - F(x_n)]u(1)$$

where

$$F(x^-) = \lim_{h \rightarrow 0} F(x-h) \text{ as } h \rightarrow 0 \text{ from positive values,} \quad (\text{A-8})$$

and

$$F(1^-) = \lim_{h \rightarrow 0} (F(1-h)) \text{ as } h \rightarrow 0 \text{ from positive values.} \quad (\text{A-9})$$

Since F is a right continuous step function, (A-7) can be rewritten as

$$\begin{aligned} E_F[u(x)] &= F(x_0)u(x_0) + \sum_{i=1}^n [F(x_i) - F(x_{i-1})]u(x_i) + [1 - F(x_n)]u(1) \\ &= - \sum_{i=1}^n F(x_{i-1}) [u(x_i) - u(x_{i-1})] + u(1) \end{aligned} \quad (\text{A-10})$$

Since

$$u(x_i) - u(x_{i-1}) = \int_{x_{i-1}}^{x_i} u'(x) dx \quad (\text{A-11})$$

Equation (A-10) can be rewritten as

$$\begin{aligned} E_F[u(x)] &= u(1) - \sum_{i=1}^n F(x_{i-1}) \int_{x_{i-1}}^{x_i} u'(x) dx \\ &= u(1) - \int_0^1 F(x) u'(x) dx \end{aligned} \quad (A-12)$$

Now form a similar expression for distribution G and subtract from (A-12) which gives

$$\begin{aligned} E_F[u(x)] - E_G[u(x)] &= \int_0^1 [G(x) - F(x)] u'(x) dx \\ &= \int_0^1 D^1(x) u'(x) dx \end{aligned} \quad (A-13)$$

where $D^1(x) = G(x) - F(x)$. Thus if $D^1(x) \geq 0$ for all $x \in I$ and the strict inequality holds for at least one value of x then $E[u(x)_F] > E[u(x)_G]$ for all $u \in U_1$ and proves the sufficiency part of Theorem (A-1) for the case $I = [0,1]$.

To examine the case U_2 , integrate (A-13) by parts:

$$\begin{aligned} E[u(x)_F] - E[u(x)_G] &= \int_0^1 u^1(x) dD^2(x) = D^2(1)u'(1) \\ &\quad - \int_0^1 D^2(x)u''(x)dx \end{aligned} \quad (A-14)$$

For $u \in U_2$, $u'(1) \geq 0$ and $u'' < 0$ on $I = (0, 1)$ which proves sufficiency in Theorem (A-2).

To examine U_3 , rewrite (A-14) for a linear utility function $u(x) = x$. In this case, $D^2(1) = \mu_F - \mu_G = \int_0^1 [G(x) - F(x)] dx$. Thus, (A-14) can be rewritten as

$$E[u(x)_F] - E[u(x)_G] = (\mu_F - \mu_G)u'(1) - \int_0^1 D^2(x)u''(x)dx \quad (A-15)$$

Integrating (A-15) by parts:

$$\begin{aligned} E[u(x)_F] - E[u(x)_G] &= (\mu_F - \mu_G) u'(1) - D^3(1)u''(1) + \\ &+ \int_0^1 D^3(x)u'''(x)dx \end{aligned} \quad (A-16)$$

For $u \in U_3$ we have $u'(1) \geq 0$, $u''(1) \leq 0$ and $u''' > 0$ on I° which proves sufficiency in Theorem (A-3).

Next consider the unbounded case $I = (0, \infty)$. Defining

$$u(\infty) = \lim_{x \rightarrow \infty} u(x) \quad (A-17)$$

which is finite by assumption, we have

$$E[u(x)_F] = u(\infty) - \int_0^\infty F(x)u'(x)dx. \quad (A-18)$$

To prove this, let $x_0 > 0$ be a continuity point of F , and assume without loss of generality that $u(0) = 0$, so that $u(x) > 0$ on I° . Then

$$\begin{aligned} E[u(x)_F] &= \int_0^{x_0} u(x)dF(x) + \int_{x_0}^\infty u(x)dF(x) \\ &= u(x_0)F(x_0) - \int_0^{x_0} F(x)u'(x)dx + \int_{x_0}^\infty u(x)dF(x). \end{aligned} \quad (A-19)$$

Now

$$0 \leq \int_{x_0}^\infty u(x)dF(x) \leq u(\infty) \int_{x_0}^\infty dF(x) = u(\infty) [1 - F(x_0)], \quad (A-20)$$

which approaches 0 as $x_0 \rightarrow \infty$. Similarly,

$$\begin{aligned} 0 &\leq \int_0^{x_0} F(x)u'(x)dx - \int_0^{x_0} F(x)u'(x)dx = \int_{x_0}^\infty F(x)u'(x)dx \\ &\leq \int_{x_0}^\infty u'(x)dx = u(\infty) - u(x_0), \end{aligned} \quad (A-21)$$

which approaches 0 as $x_0 \rightarrow \infty$. Thus, as x_0 approaches infinity, we get (A-18). Therefore,

$$E[u(x)_F] - E[u(x)_G] = \int_0^{\infty} D^1(x)u'(x)dx \quad (\text{A-22})$$

which is the same as for the bounded case. We need only note that the boundedness of U on I implies that u' , u'' , and u''' all approach zero at ∞ so that integration of (A-22) by parts, as in the finite case, is valid. Theorems (A-1) through (A-3) then follow in a fashion similar to the bounded case for the unbounded case and the sufficiency conditions for Theorems (A-1) through (A-3) are met.

The necessary parts of Theorems (A-1) through (A-3) can be proved by example. Consider first the subclass of utility functions belonging to U_1 and suppose that $D^1(x) < 0$ for some $x \in I$. Then F and G must cross, that is, there must be a point of continuity, x_0 , of both F and G in the interval I . We know then that

$$D^1(x_0) = [1 - F(x_0)] - [1 - G(x_0)] = E_F(f_{x_0}(x)) - E_G(f_{x_0}(x)) \quad (\text{A-23})$$

where f_{x_0} is defined as the step function

$$f_{x_0}(x) = \begin{cases} 1, & x > x_0 \\ 1/2, & x = x_0 \\ 0, & x < x_0. \end{cases} \quad (\text{A-24})$$

¹Since x_0 is a point of continuity of F and G , the value of $F_{x_0}(x_0)$ is immaterial.

Note that f_{x_0} is a nondecreasing function but is not a member of U_1 . However, it is possible to approximate F_{x_0} arbitrarily closely by a function which is a member of U_1 , so there exists a function belonging to U_1 such that $E_F[u(x)] < E_G[u(x)]$. For example, take $U_n(x) = 1/2 + 1/2 (x-x_0)[(x-x_0)^2 + 1/n^2]^{-1/2}$. For $n > 0$ $u_n \in U_1$ and as $n \rightarrow \infty$, $u_n(x) \downarrow 0$ for $x < x_0$, $u_n(x) \uparrow 1$ for $x > x_0$, and $u_n(x_0) = 1/2$ for all n . Applying the Monotone or the Dominated Convergence Theorem (38 and 39) on the intervals $In \ x:x \leq x_0$ and $In \ x:x \geq x_0$, we have:

$$E_F[u_n(x)] \rightarrow E_F[f_{x_0}(x)] \text{ as } n \rightarrow \infty \quad (\text{A-25})$$

A similar case can be constructed for distribution G . Thus $D^1(x_0) < 0$ implies $E_F[U_n(x)] < E_G[u_n(x)]$ for some n , so that $F >_1 G$ can not be true which proves the necessary part of the proof for first degree stochastic dominance.

In the context of second degree stochastic dominance, suppose $D^2(x_0) < 0$ for some $x_0 \in I$ where x_0 is a point of continuity between F and G . Integrating by parts gives:

$$\begin{aligned} D^2(x_0) &= \int_0^{x_0} [G(x) - F(x)]dx = \int_0^{x_0} x[dF(x) - dG(x)] + \\ &\quad x_0[G(x_0) - F(x_0)] \\ &= \int_0^{x_0} (x-x_0)[dF(x) - dG(x)] = E_F[g_{x_0}(x)] - E_G[g_{x_0}(x)] \quad (\text{A-26}) \end{aligned}$$

where $g_{x_0}(x)$ is a piecewise linear ramp function.

$$g_{x_0}(x) = \begin{cases} x-x_0, & x \leq x_0 \\ 0, & x > x_0 \end{cases} \quad (\text{A-27})$$

The function $g_{x_0}(x)$ is nondecreasing and concave but is not a member of U_2 . However, the functions $v_n(x) = 1/2(x-x_0) - 1/2[(x-x_0)^2 + 1/n^2]^{1/2}$ are in I_2 , and $v_n(x) \rightarrow g_{x_0}(x)$ as $n \rightarrow \infty$. By invoking the Dominated Convergence Theorem, $D^2(x) < 0$ implies $E_F[v_n(x)] < E_G[v_n(x)]$ for large n , so $F >_2 G$ is false which proves the necessary part of second degree stochastic dominance condition in (A-2).

To prove the necessary part of (A-3), two conditions must be established: $\mu_F \geq \mu_G$ and $D^3(x) \geq 0$. Again, suppose $D^3(x) < 0$ for some point of continuity x_0 . Integrating by parts

$$\begin{aligned} D^3(x_0) &= \int_0^{x_0} D^2(x)dx = x_0 D^2(x_0) - \int_0^{x_0} D^1(x)xdx \\ &= -x_0 \int_0^{x_0} (x-x_0)dD^1(x) - 1/2x_0^2 \int_0^{x_0} dD^1(x) + 1/2 \int_0^{x_0} x^2 dD^1(x) \\ &= 1/2 \int_0^{x_0} (x-x_0)^2 dD^1(x) = 1/2 \int_0^{x_0} [-(x-x_0)^2][dF(x)-dG(x)] \\ &= E_F[h_{x_0}(x)] - E_G[h_{x_0}(x)]. \end{aligned} \quad (\text{A-28})$$

where h_{x_0} is the piecewise quadratic function

$$h_{x_0}(x) = \begin{cases} -1/2(x-x_0)^2, & x \leq x_0 \\ 0, & x > x_0 \end{cases} \quad (\text{A-29})$$

Again, although $h_{x_0}(x)$ does not belong to U_3 , the functions $w_n(x) = 1/2 \int_x^{x_0} \{[(y-x_0)^2 + 1/n^2]^{1/2} - (y-x_0)\} dy$ do belong to U_3 for $n > 0$, and $w_n(x) \rightarrow h_{x_0}(x)$ as $n \rightarrow \infty$. Thus, $D^3(x_0) < 0$ implies $E_F[w_n(x)] < E_G[w_n(x)]$ for large n and $F >_3 G$ is false.

Finally, suppose $\mu_F < \mu_G$. Consider the function $u(x) = e^{-kx}$ ($k > 0$), which is in U_3 . Define $\phi_k(x) = kx - 1 + e^{-kx}$, and note that $0 \leq \phi_k(x)/k \leq x$ for all $x \geq 0$. Also $\phi_k(x)/k \rightarrow 0$ as $k \rightarrow 0$, for all, $x \geq 0$. The dominated convergence Theorem thus implies

$$\lim_{k \rightarrow 0} \int_I [\phi_k(x)/k] dF(x) = 0 \quad (\text{A-30})$$

$$\text{so } \int_I \phi_k(x) dF(x) = o(k) \quad (\text{A-31})$$

Thus $E_F[u(x)] = -1 + k\mu_F + o(k)$ as $k \downarrow 0$ and similarly for $E_G[u(x)]$. Then $E_F[u(x)] < E_G[u(x)]$ for sufficiently small k , and $F >_3 G$ is false.

APPENDIX B:

ESTIMATED COEFFICIENTS AND STATISTICAL RESULTS

This appendix presents the results of the statistical regressions needed to run the simulation model described in (5-1) through (5-33). Estimated coefficients, t-values, R^2 and other relevant statistics are presented for the projected 1980 balance sheets of the representative farms, net operating incomes, cash fixed operating expenses, noncash fixed operating expenses and the parameters of the Monte Carlo triangular distribution.

Projected 1980 Balance Sheet

The projected 1980 Balance Sheets for the representative farms which are summarized in Table 6 were estimated from the cross-sectional, time-series of Iowa Farm Business Associations Annual Surveys for the years 1970 through 1979 (63). The model used was of the form

$$Y_t = X_t \beta + \mu_t \text{ and} \quad (B-1)$$

$$\mu_t = \rho \mu_{t-1} + e_t,$$

where

Y_t is the category being estimated,

t is for the years 1970 through 1979,

X_t is a vector including an intercept term and the year, and

e_t and μ_t are assumed to satisfy the standard assumptions of

$$e_t \sim (0, \sigma^2),$$

$$e_t, e_j = 0 \text{ for } t \neq j,$$

$$|\rho| < 1,$$

$$E(\mu_t) = 0$$

$$V(\mu_t) = \frac{\sigma^2}{1-\rho^2}, \text{ and}$$

$$\text{COV}\{u_t, u_{t-h}\} = \frac{\rho^h}{1-\rho^2} \sigma^2.$$

The coefficients, t-values and R^2 are summarized in Table B-1 for the five representative farms. The estimated 1980 liabilities for 1980 from Table B-1 were adjusted upward by the debt to asset ratios of Table 6 to arrive at the liability values appearing in Table 6.

Income and Fixed Expense Projections

Estimated net operating income of (5-2) was estimated using the model of (B-1) for each class of farm. The $\hat{\alpha}_{ji}$ coefficients with t-values, lag coefficient and R^2 are summarized in Table B-2.

Cash fixed operating costs of (5-4) and noncash fixed operating costs of (5-5) were also estimated using the one period lag model of (B-1). The $\hat{\beta}_{ji}$ and $\hat{\gamma}_{ji}$ coefficients, t-values, lag coefficient and R^2 are summarized in Tables B-3 and B-4 respectively.

The coefficients in Tables B-2, B-3 and B-4 predict well as evidenced by the high R^2 , but most t-values are not significant. This phenomenon is due to the high degree of collinearity among the independent variables. In almost all cases, the correlation coefficients between the asset types exceeded .9. Fortunately, this problem is not as severe as it first appears. If the representative farm is assumed to

already be on its expansion path and that path is linear through the origin, then expansion or contraction of the firm size will occur only by increasing or decreasing the asset categories in equal proportions. That is, no substitution of one asset for another takes place nor is it possible to increase income by increasing one asset category without a proportionate increase in the others. Therefore, since all independent variables (except the year and the intercept) are changed proportionately, the effects of the collinearity are mitigated.

Parameters of the Monte Carlo Distribution

As discussed in Chapter V on the Monte Carlo simulation, the triangular distribution requires the specification of a lower limit, a mode and an upper limit. From the Iowa Farm Business Association's annual surveys for the years 1970 through 1979, the mean, standard deviation, skewness, kurtosis and coefficient of variation were estimated for each representative farm. These statistics are summarized in Table B-5. Although the sample sizes are too small to perform statistical tests of significance, the values presented in Table B-5 indicate that classes 1, 2 and 4 are slightly negatively skewed while classes 3 and 5 are positively skewed. Furthermore, class 1 farms evidence platykurtic behavior (relative to a normal distribution) while classes 2 through 5 evidence increasingly leptokurtic behavior (i.e., less peakedness and thicker tails than a normal distribution). It is interesting to note that for classes 1 through 4 the absolute dispersion increases with

increasing levels of income but, as measured by the coefficient of variation, it does so at a decreasing rate.

The information of Table B-5 can be used to construct points A, M and B of the triangular distribution for each class of farm. For a farm of class i , the lower bound A is

$$\bar{X}_i A_i = \bar{X} - P_i \cdot SD_i, \quad (B-2)$$

where

\bar{X}_i is the mean of class i from Table B-5,

A_i is the lower bound,

P_i is the two-tailed probability measured in standard deviations about the mean, and

SD_i is the standard deviation of class i from Table B-5.

Similarly the upper limit of the triangular distribution for class i is

$$\bar{X}_i B_i = \bar{X} + P_i \cdot SD_i, \quad (B-3)$$

where B_i is the upper bound for class i farms. Dividing (B-2) and

(B-3) through by the mean produces

$$A_i = 1 - P \cdot \frac{SD_i}{\bar{X}_i} = 1 - P \cdot CV_i, \text{ and} \quad (B-4)$$

$$B_i = 1 + P \cdot \frac{SD_i}{\bar{X}_i} = 1 + P \cdot CV_i,$$

where CV_i is the coefficient of variation for class i .

The value of P is set in accordance with a normal distribution. If the underlying distribution is normal, then the triangular distribution

can be set to account for 95 percent of that distribution by setting P equal to 1.96 standard deviations about the mean. (With $P=3.00$, 99% of the normal distribution would be included in the triangular.) But as was pointed out above, none of the distributions appear normal. Therefore, by setting P equal to 1.96, something less than 95 percent of the distribution is accounted for in classes 2 through 5 because of their leptokurtic behavior. Conversely, since class one appears platykurtic, more than 95 percent of the distribution is included in the triangular distribution.

The values of A, M and B for each representative farm are summarized in Table B-6 with P set at 1.96 standard deviations about the mean. The parameters of Table B-6 are employed in (5-39) to estimate the randomly generated error term to adjust the predicted net operating income to an actual net operating income in (5-3).

Table B-1. Estimated coefficients, t-values and R² for 1980 projected balance sheets for the representative farms

	I			II		
	Intercept	Year	R ²	Intercept	Year	R ²
Current Assets	-347.704 (-2.65)	5,411.7 (3.14)	.66	-329,892 (-2.16)	5,586.5 (2.78)	.61
Intermediate Assets	-441,191 (-14.724)	6,418.1 (16.284)	.98	-693,342 (-9.75)	9,931.4 (10.62)	.96
Long-Term Assets	-1,053,742 (-7.36)	15,455 (8.21)	.93	-1,683,405 (-9.88)	24,247.8 (10.82)	.96
Current Liabilities	-17,658 (-1.29)	293.4 (1.64)	.35	5,948.9 (.176)	28.0 (.063)	.0008
Intermediate Liabilities	-11,695.2 (-1.81)	176.0 (2.07)	.46	-89,809 (-6.32)	1,241.8 (6.64)	.90
Long-Term Liabilities	-35,831.0 (-1.18)	538.8 (1.35)	.27	-85,391 (-5.58)	1,230.8 (6.11)	.88

Table B-1. continued

	III			IV		
	Intercept	Year	R ²	Intercept	Year	R ²
Current Assets	-755,247 (-11.08)	11,775.7 (13.13)	.97	-726,433 (-6.06)	11,813.5 (7.50)	.92
Intermediate Assets	-958,691 (-20.73)	13,728.3 (22.56)	.99	-908.502 (-38.15)	13,205.4 (42.16)	.997
Long-Term Assets	-2,152,056 (9.47)	31,167.5 (10.43)	.96	-2,912,000 (-11.53)	42,002.7 (12.65)	.97
Current Liabilities	-94,782 (-2.94)	1,414.6 (3.33)	.69	-25,680 (-0.72)	543.7 (1.17)	.21
Intermediate Liabilities	-71,654 (-7.24)	1,011.9 (7.77)	.92	-70,198 (-2.90)	1,010.1 (3.17)	.67
Long-Term Liabilities	-79,050 (-3.28)	1,224.6 (3.86)	.75	-114,720 (-4.14)	1,739.9 (4.78)	.82

Table B-1. continued

	-----V-----		R ²
	Intercept	Year	
Current Assets	-1,351,370 (-5.12)	21,609 (6.23)	.89
Intermediate Assets	-1,520.730 (-30.92)	22,098.4 (34.16)	.996
Long-Term Assets	-4,961,422 (-13.16)	71,525.6 (14.43)	.98
Current Liabilities	-197,131 (-3.61)	2,939.0 (4.10)	.77
Intermediate Liabilities	-137,527 (-7.10)	1,931.4 (7.55)	.92
Long-Term Liabilities	-390,753 (-3.56)	5,569.8 (3.86)	.75

Table B-2. Net operating income (NOI) by class. Coefficients, t-values in parentheses, lag coefficients and R^2 are estimated with a one period lag autoregressive model

Class	$(\hat{\alpha}_{01})$ 1	$(\hat{\alpha}_{21})$ CA	$(\hat{\alpha}_{31})$ IA	$(\hat{\alpha}_{41})$ FA	Year	ρ	R^2	Rotated Acres
1	-741,120.7 (-.29)	-2.685 (-.74)	8.218 (1.52)	-2.381 (-.93)	12,432.9 (.321)	.4806 (1.45)	.93	136
2	875,141.6 (.36)	.080 (.043)	2.469 (.67)	-.351 (-.98)	-12,028.7 (-.36)	.4275 (1.25)	.87	196
3	-276,301.8 (-.32)	-.178 (-.14)	1.618 (1.14)	-.554 (-2.38)	5,038.8 (.387)	.6099 (2.04)	.92	282
4	-340,475.3 (-.12)	-4.271 (-2.36)	10.194 (7.00)	-1.631 (-2.77)	10,337.4 (.23)	.6535 (2.28)	.99	359
5	25,043,417.8 (2.26)	1.478 (1.03)	11.149 (4.83)	1.025 (1.12)	-370,217.7 (-2.24)	.4247 (1.24)	.99	614

Table B-3. Cash fixed operating costs (CFOC) by class. Coefficients, t-values in parentheses, one period lag coefficient and R^2 are estimated with an autoregressive model

Class	$(\hat{\beta}_{oi})$ 1	CA	$(\hat{\beta}_{1i})$ IA	$(\hat{\beta}_{2i})$ FA	$(\hat{\beta}_{3i})$ Year	ρ	R^2
1	-5,070.4 (-1.06)		.071 (7.00)	-.0143 (-8.88)	97.45 (1.40)	.5297 (1.65)	.99
2	31,658.7 (3.22)		.045 (2.73)	.0098 (1.79)	-425.56 (-3.01)	.6728 (2.41)	.99
3	-216,595.3 (-.99)		.089 (.38)	-.0968 (-2.62)	3,177.14 (1.01)	.6506 (2.27)	.76
4	-67,018.6 (-1.00)		-.016 (-.27)	-.0053 (-1.25)	1,015.61 (1.05)	.4880 (1.48)	.99
5	365,010.7 (-1.36)		-.079 (-.63)	-.0285 (-2.35)	5,273.47 (1.38)	.4835 (1.46)	.98

Table B-4. Depreciation (noncash fixed operating costs, NCFC) by class. Coefficients, t-values in parentheses, one period lag coefficient and R^2 are estimated with an autoregressive model

Class	(\hat{Y}_{0i}) 1	CA	(\hat{Y}_{1i}) IA	(\hat{Y}_{2i}) FA	(\hat{Y}_{3i}) Year	ρ	R^2
1	14,517.62 (.70)		.105 (2.26)	-.0015 (-.23)	-168.91 (-.56)	.3440 (.97)	.98
2	38,223.1 (1.83)		.082 (2.38)	-.0188 (1.72)	-519.27 (-1.73)	.4945 (1.51)	.99
3	-61,005.3 (-2.55)		-.007 (.28)	-.0053 (1.20)	921.01 (2.69)	.5216 (1.62)	.99
4	-161,959.5 (-3.20)		-.038 (-.85)	-.0113 (-3.49)	2,390.70 (3.26)	.2893 (.80)	.99
5	-503,461.3 (-4.15)		-.163 (-2.89)	-.020 (-3.74)	7,275.32 (4.20)	.6107 (2.04)	.99

Table B-5. Statistical measures of net operating income from the Iowa Farm Business Association annual surveys by Class for the years 1970 through 1979

Class	Mean (X)	Standard Deviation (S.D.)	Skewness (γ_1)*	Kurtosis (γ_2 **	Coefficient of variation (CV)***
1	\$ 33,284	\$14,659	- .354	-2.507	.44043
2	45,900	15,645	- .267	.976	.34084
3	59,463	18,318	1.203	1.575	.30805
4	73,335	21,530	- .368	2.038	.29358
5	123,033	58,633	1.955	4.528	.47656

$$*\gamma_1 = \frac{\Sigma(x-\bar{x})^3/n}{[\Sigma(x-\bar{x})^2/n]^{3/2}}$$

$$**\gamma_2 = \frac{\Sigma(x-\bar{x})^4/n}{[\Sigma(x-\bar{x})^2/n]^2} - 3$$

$$***CV = \frac{\sqrt{\Sigma(x-\bar{x})^2/n}}{\bar{x}} \times 100$$

Table B-6. Parameters of the triangular distribution with a probability of ninety-five percent ($P=.95$)

	Lower Limit	Upper Limit	Mode	Mean
Class	A	B	M	\bar{X}
1	0.1368	1.8632	0.85	1
2	0.3320	1.6680	0.90	1
3	0.3962	1.6038	1.10	1
4	0.4246	1.5754	0.80	1
5	0.0659	1.9341	1.30	1

APPENDIX C:

**THE IOWA STATE UNIVERSITY BUSINESS AND
FINANCIAL PLANNING MODEL**

The Iowa State University Business and Financial Planning Model is composed of 30 subroutines. The program is written in PL/1 programming language and encompasses over 9,000 programming statements. The primary objective of the model is to serve as a computer aid in the planning stage of the decision making process. As such, the model identifies the financial consequences for a given set of production, investment and financial decisions. Also, the model compares the financial consequences of the sole proprietorship, partnership and regularly taxed corporation forms of legal organization.

The necessary input for the model is in three parts. The first part details the personal characteristics of each family and nonfamily member involved with the business. The second part describes the current balance sheet of the farm and the personal assets and liabilities of each individual. The third part of the input details the ownership characteristics and income sharing arrangements of the family firm if it is organized as a sole proprietorship, a partnership or a corporation.

Exogenous parameters supplied to the model include the annual anticipated rates of inflation, interest rates and rates of return on business and personal property over the specified planning horizon.

For each legal form of organization, the model generates a set of expected comparative financial statements. These comparative statements include a comparative statement of financial position, a comparative statement of changes in financial position (incorporating a statement of

income) and a comparative statement of cash flows for each family member.

The following discussion identifies each subroutine in the model and briefly describes the subroutines functions.

MAIN

The main subroutine identifies all the variables, working matrices and output vectors to be used in the model. It controls the flow of execution through the model and calls the supporting subroutines in the correct sequence. The calculations are performed iteratively for each year of the planning horizon. After all computations are completed, the report generating subroutines are called.

The subroutines called by MAIN and the order in which they are called are:

1. DATE
2. INPUT
3. OUTAA
4. CARLO
5. CLEAN
6. FRM ASS
7. ALLOCTE
8. PLANS
9. INFLTN
10. SOLE
11. PRTNERS
12. CORP_C
13. CORP_S
14. FLOWS
15. FRM_INV
16. SUMARY
17. OUTCC
18. OUTBB
19. OUTEE

ALLOCTE

At the beginning of each planning period, this subroutine determines the value of each owner's equity interest in the firm. Assets and liabilities are valued at fair market values.

The only subroutine called by ALLOCTE is PER_ASS.

BUY

This subroutine reinvests any positive cash flows after consumption and living expenses in new business or personal assets. For the sole proprietorship it actually acquires the assets. For the partnership or corporation, it merely identifies the amount available for investment.

Subroutines called by BUY are:

1. PER_ASS
2. UPDATE

CAP_ASS

This procedure calculates the taxable, deductible and nontaxable portions of all capital transactions and any other transactions which qualify for capital gain treatment under Section 1231 of the Internal Revenue Code. This subroutine also calculates ordinary income and loss on capital transactions along with short and long term carrybacks and carryforwards.

CARLO

This subroutine generates a stochastic variate in the range (0, 1) using a power residual random number generator. The variate is mapped

into the cumulative density function of a triangular distribution with preassigned parameters to determine the Monte Carlo error term to be applied to predicted net operating income.

CLEAN

This subroutine initializes to zero all working matrices, vectors and variables at the start of the planning horizon for each legal form of organization.

CONSUMP

This subroutine estimates the living expenses for each family and nonfamily member.

CORP_C

This subroutine calculates the financial and tax consequences for the regularly taxed corporation. Calculations performed include the determination of gross income, net taxable income, fixed operating costs, depreciation, debt servicing, salaries and director's fees, capital gain or loss (including carryover), charitable deductions and any net operating loss. The regular corporate tax is calculated and if net long term capital gains exceed net short term capital losses, the tax under the alternate method is also calculated. Next, the lesser of the regular tax or the alternate tax is compared to the minimum tax on tax preference

items to find which is larger. This value becomes the corporate federal income tax.

To the extent possible, dividends on common and preferred stock are paid from earnings and profits in accordance with the specified income sharing plan.

Finally, any accumulated earnings tax liability is calculated.

Subroutines called by CORP_C and the order in which they are called are:

1. DEPREC
2. RTRE_DT
3. CAP_ASS

DATE

This is a generic, built in subroutine which returns the current day, month and year.

DEPREC

This subroutine computes the per period and accumulated depreciation expenses for the asset base described in the financial statement.

FLows

This subroutine calculates the per period cash flow for each individual. Salaries, director's fees, nonfarm income, social security, interest income and dividends are all included in personal income. Personal income taxes, charitable and noncharitable gifts and personal consumption expenditures are calculated.

Any excess positive cash flow is available for reinvestment in business and personal assets. Any deficit cash flow is made up with short term personal borrowings.

Subroutines called by FLOWS are:

1. RTRE_DT
2. CONSUMP
3. CAP_ASS
4. INC_TAX
5. PER_ASS
6. UPDATE

FRM_ASS

This subroutine totals the assets and liabilities by maturities.

FRM_INV

This subroutine invests (or disinvests) in new (existing) assets in the same proportion as the existing ownership pattern.

The only subroutine called by FRM_INV is UPDAT2.

INC_TAX

This subroutine determines the personal federal income tax liability for each member. It calculates gross income, adjusted gross income, deductions for dependents, itemized deductions and the tax liability from the appropriate table.

The only subroutine called by INC_TAX is CAP_ASS.

INFLTN

This subroutine determines the appreciation in asset nominal values due to the effects of inflation. The inflation rates by asset type are exogenously specified.

INPT

This subroutine reads and stores the input for the family characteristics, financial statement, ownership pattern and anticipated financial and transaction plans for the particular situation to be analyzed.

INT_ROR

The subroutine interactively solves for the internal rate of return on the net change in nominal equity ownership of an individual over the planning horizon.

OUTAA

This subroutine outputs the family characteristics, property inventory and initial financial statements for each legal form of organization.

OUTBB

This subroutine outputs the summary comparison figures for the sole proprietorship, partnership and regularly taxed corporation.

OUTCC

This subroutine outputs the comparative financial statements of position, changes in position and individual cash flows for each legal form of organization.

OUTEE

This subroutine outputs the first, second and third degree stochastic dominance discrete cumulative density values for total net assets owned by the parents and each heir.

PER_ASS

This subroutine calculates the net fair market value of the personal assets owned by an individual.

PLANS

This subroutine handles all the anticipated transactions such as gifts, land sales, asset purchases, setting up trusts, long term debt borrowing and long term debt prepayment. The subroutine incorporates the anticipated (planned) transaction at the correct time into the flow of events.

Subroutines called by PLANS listed in the order in which they are called are:

1. PER ASS
2. UPDATE

PRTNERS

This subroutine determines distributable income for the partnership and makes the distribution in accordance with the prior agreement. If there is no prior agreement specified, distribution is made in accordance with ownership interests.

Subroutines called by PRTNERS are:

1. DEPREC
2. RTRE_DT

RTRE_DT

This subroutine calculates the current interest expense and principal payments due on outstanding liabilities for both the firm and each individual.

SOLE

This subroutine calculates for the sole proprietorship the income and expenses from operations and distributes the earnings in relation to asset ownership.

Subroutines called by SOLE are:

1. DEPREC
2. RTRE_DT

CORP_S

This subroutine details the financial consequences of the specially taxed subchapter S corporation.

SUMARY

This subroutine calculates and saves the summary comparison statistics among the legal forms of organization.

Subroutines called by SUMARY are:

1. INC_TAX
2. INT_ROR

UPDATE

This subroutine updates the property inventory and ownership information for purchases, sales and transfers of individuals and the sole proprietorship.

UPDAT2

This subroutine updates the property inventory and the ownership information for the partnership, regularly taxed corporation and subchapter S corporation.

APPENDIX D:

TESTS FOR SIGNIFICANT DIFFERENCES BETWEEN MEANS

Stochastic dominance theorems are very effective in distinguishing between two distributions. Although the theorems can identify the preferred distributions, they provide no information on how much more preferred one distribution is over another. In several of the figures of Chapter VI, the curves are so close together as to intuitively suggest that, as a practical matter, little significant difference exists between the financing arrangements. This intuitive reasoning is supported by two arguments. First, each cumulative density function is based on a probability distribution generated from a random sample of a population for which the statistical moments are unknown. This implies that sampling error could explain the difference between the cumulative density functions. Second, even if two cumulative density functions are significantly different, the monetary difference may not be large enough to justify altering the present financing arrangements.

The latter argument can not be addressed without more knowledge about a decision maker's utility specification and the costs of altering the present organizational and financing structure to the preferred structure. However, the first argument can be addressed.

Any two cumulative density functions created from two independent samples have means \bar{X}_1 , \bar{X}_2 , which are estimates of their respective population means μ_1 , μ_2 .¹ Testing for a significant difference between

¹See Snedecor (114A, pp. 100-116).

the population means, $\mu_1 - \mu_2$, is based on the t-distribution where t has the value

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} . \quad (D-1)$$

The denominator is the variance of the difference of two populations which equals the sum of the variances. However, the population variances are unknown. The two samples furnish unbiased estimates S_1^2 of σ_1^2 and S_2^2 of σ_2^2 . Consequently, the ordinary t of (D-1) can be replaced by the quantity.

$$t' = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} . \quad (D-2)$$

When the number of observations are the same for each sample ($n_1 = n_2 = n$), Equation (D-2) simplifies to

$$t' = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2) \quad n}{S_1^2 + S_2^2} . \quad (D-3)$$

To test the null hypothesis that $\mu_1 = \mu_2$, the standard normal deviate, Z, can be constructed from (D-3), or

$$Z = \frac{n (\bar{X}_1 - \bar{X}_2)}{S_1^2 + S_2^2} . \quad (D-4)$$

From a cumulative normal frequency distribution, the probability of a larger value of Z occurring, ignoring sign, can be determined. Table D-1 presents the mean, variance, skewness and kurtosis for selected cumulative density functions in Chapter VI. In addition, the calculated Zs, the probability of a larger value occurring and mean differences significant at the 80 and 95 percent levels are presented.

Table D-1. Tests of significant differences between means of the probability distribution functions of the cumulative density functions of Chapter VI

Figure from Chapter VI	Mean (x \$1,000)	Variance (x \$1,000,000)	Skewness	Kurtosis	Null Hypothesis	Z
38						
A. Salaries	1,807.6	17,187.2	-.604	.240	$H_0: \mu_A = \mu_B$	12.61**
B. Dividends	1,518.1	3,881.3	-.985	.915		
39						
A. Salaries	19.6	4.2	-.955	.966	$H_0: \mu_A = \mu_B$	31.74**
B. Dividends	8.38	.799	1.20	-.164		
40						
A. Salaries	1,102.5	6,416.0	-.621	.274	$H_0: \mu_A = \mu_B$	15.14**
B. Dividends	893.5	1,211.0	-.867	1.82		
41						
A. Salaries	726.5	2,756.3	-.615	.261	$H_0: \mu_A = \mu_B$	12.20**
B. Dividends	613.1	702.3	-.960	2.12		
42						
A. All Common	3,781.5	252,506.3	-2.79	.448	$H_0: \mu_A = \mu_B$: $\mu_B = \mu_C$: $\mu_A = \mu_C$.88
B. Half Loan	3,883.5	285,262.8	.383	.079		1.65*
C. Half Bond	4,084.0	306,583.7	.384	.085		2.56**
44						
A. Half Bond	877.6	10,363.2	.424	.175	$H_0: \mu_A = \mu_B$: $\mu_B = \mu_C$: $\mu_A = \mu_C$	6.32**
B. All Common	1,048.2	18,796.4	-.292	.462		1.31*
C. Half Loan	1,085.9	14,280.3	.448	.242		8.39**
46						
A. All Common	2,936.7	150,621.6	-.284	.453	$H_0: \mu_A = \mu_B$: $\mu_B = \mu_C$: $\mu_A = \mu_C$.59
B. Half Loan	2,990.0	181,220.5	-.393	.607		3.39**
C. Half Bond	3,323.6	212,890.0	-.411	.670		4.06**

Table D-1. continued

Figure from Chapter VI	Mean (x \$1,000)	Variance (x \$1,000,000)	Skew- ness	Kurtosis	Null Hypothesis	Z
49						
A. Half Bond	1,609.5	37,908.1	-.354	.630	$H_0: \mu_A = \mu_B$	5.64**
B. All Common	1,893.1	63,302.6	-.280	.449	: $\mu_B = \mu_C$	1.34*
C. Half Loan	1,964.8	50,580.0	-.299	.538	: $\mu_A = \mu_C$	7.55**
52						
A. All Common	3,781.5	252,506.3	-.279	.448	$H_0: \mu_A = \mu_B$	1.23
B. Half Loan	3,924.4	287,296.0	.389	.085	: $\mu_B = \mu_C$	1.83*
C. Half Bond	4,147.4	308,913.6	.393	.096	: $\mu_C = \mu_A$	3.09**
53						
A. Half Bond	838.2	9,722.0	.421	.160	$H_0: \mu_A = \mu_B$	7.86**
B. All Common	1,048.2	18,796.4	-.292	.462	: $\mu_B = \mu_C$.38
C. Half Loan	1,059.0	13,689.0	.447	.234	: $\mu_A = \mu_C$	9.13**
55						
A. 12%	3,883.5	285,262.8	.383	.079	$H_0: \mu_A = \mu_B$.34
B. 9%	3,924.4	287,296.0	.389	.085		
56						
A. 9%	1,059.0	13,689.0	.447	.234	$H_0: \mu_A = \mu_B$	1.02
B. 12%	1,085.9	14,280.3	.448	.242		
57						
A. 12%	2,990.0	181,220.5	-.393	.607	$H_0: \mu_A = \mu_B$.69
B. 9%	3,056.0	183,440.9	-.369	.570		
58						
A. 9%	1,919.3	48,180.3	-.284	.508	$H_0: \mu_A = \mu_B$.92
B. 12%	1,964.8	50,580.0	-.299	.538		

Table D-1. continued

Figure from Chapter VI	Mean (x \$1,000)	Variance (x \$1,000,000)	Skew- ness	Kurtosis	Null Hypothesis	Z
59						
A. 12%	3,323.6	212,890.0	-.411	.670	$H_0: \mu_A = \mu_B$	1.01
B. 9%	3,428.1	215,110.4	-.374	.602		
60						
A. 9%	1,546.9	35,231.3	-.328	.564	$H_0: \mu_A = \mu_B$	1.46*
B. 12%	1,609.5	37,908.1	-.355	.630		

*Significant at the 80% confidence level. That is, there is 20 percent or less chance of a larger difference occurring.

**Significant at the 95% confidence level. There is five percent or less chance of a larger difference occurring.